

Kettle Logic in Abstract Argumentation

Timotheus Kampik
Umeå University, Umeå, Sweden
tkampik@cs.umu.se

Abstract

Kettle logic is a colloquial term that describes an agent’s advancement of inconsistent arguments in order to defeat a particular claim. Intuitively, a consistent subset of the advanced arguments should exist that is at least as successful at refuting the claim as the advancement of the set of mutually inconsistent arguments. In this paper, we formalise this intuition and provide a formal analysis of kettle logic in abstract argumentation, a fundamental approach to computational argumentation, showing that all of the analysed abstract argumentation semantics (inference functions) – with the exception of naive semantics, which is considered a mere simplistic helper for the construction of other semantics – suffer from kettle logic. We also provide an approach to mitigating kettle logic under some circumstances. The key findings presented in this paper highlight that agents that apply the inference functions of abstract argumentation, are – similarly to humans – receptive to persuasion by agents who deliberately advance inconsistent and intuitively “illogical” claims. As abstract argumentation can be considered one of the most basic models of computational argumentation, this raises the question to what extent and under what circumstances kettle logic-free argumentation can and should be enforced by computational means.

1 Introduction

In human argumentation, we often see fallacious patterns. Prominent examples are classical argumentation fallacies such as arguments *ad hominem* or *ad populum*, which, instead of attacking or supporting a target argument, attack the agent who uttered the target arguments and draw support by appealing to the popular opinion, respectively [25]. One particular fallacy that may be of interest from a formal reasoning perspective is a pattern that is colloquially called *kettle logic*. It refers to the advancement of mutually inconsistent arguments in order to defeat a particular claim¹. The term *kettle logic* was presumably coined first by Derrida [17], based on the following well-known example as provided in Freud’s *Traumdeutung* (interpretation of dreams) [23]².

¹For a textbook about fallacies in informal logic that (among others) features kettle logic, see [1] and more specifically the *Kettle Logic* chapter [34].

²The present work is, besides the use of this well-known example for demonstrative purposes, not at all related to *Traumdeutung* or any other of Freud’s works.

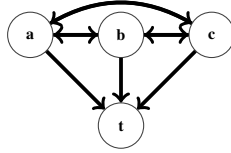


Figure 1: Freud’s classical kettle logic example, modelled as an abstract argumentation framework.

Example 1. *Person \mathcal{A} borrows a kettle from their neighbour \mathcal{B} . Eventually, \mathcal{A} returns the kettle to \mathcal{B} , whereupon \mathcal{B} notices the kettle has holes. When \mathcal{B} later complains to \mathcal{A} about the holes in the kettle (argument t), \mathcal{A} argues that:*

1. *The holes were not in the kettle when \mathcal{A} returned it to \mathcal{B} (argument a);*
2. *The holes were already in the kettle when \mathcal{A} obtained it from \mathcal{B} (argument b);*
3. *\mathcal{A} never borrowed the kettle (argument c).*

While the example is rather naive, it reflects patterns that are commonly and effectively applied, *e.g.*, in political discourse. For instance, according to social science researchers, a frequent set of arguments advanced by European right-wing populists is that to protect traditional families, legislation that facilitates the rights of homosexuals should not be advanced, while the lack of tolerance for homosexuality among certain groups of immigrants is simultaneously used as an argument to advance the idea that immigration is a threat to an open and tolerant western society. The inconsistency of these arguments (simultaneous rejection and support of the rights of homosexuals) is ignored, as long as the arguments advance predetermined political goals [29].

The prevalence of kettle logic in human argumentation serves as a motivation to formalise the underlying intuition and study its occurrence in computational argumentation. As a starting point, let us formally model Example 1 using abstract argumentation [19], in which we draw inferences from directed graphs – so-called Argumentation Frameworks (AFs), *i.e.*, tuples $F = (AR, AT)$, where AR is a set of elements (“arguments”) and AT a binary relation on AR (“attacks”). From an AF, inferences are drawn using an argumentation semantics, which yield one, several, or no sets of arguments (“extensions”) as a result.

Example 2 (Example 1 continued). *We model the classical kettle logic example as an AF $F = (\{t, a, b, c\}, \{(a, t), (a, b), (a, c), (b, t), (b, a), (b, c), (c, t), (c, a), (c, b)\})$ (Figure 1). All argumentation semantics that the formal argumentation community accepts as useful and reasonable either infer the empty set from this framework or the three extensions $\{a\}$, $\{b\}$, and $\{c\}$.*

One may reasonably claim that the use of kettle logic in Example 2 is not particularly problematic. After all, advancing only one of the arguments a , b , or c would be, in the case of many argumentation semantics, just as good for defeating t as advancing the set of inconsistent arguments a , b , and c : clearly, the AF $F' = (\{t, a\}, \{(a, t)\})$ yields only one extension, which is $\{a\}$, and hence already allows for the defeat of the

target argument t . However, we can find examples where a consistent subset of the inconsistent kettle logic arguments that reaches the goal of defeating the target argument cannot be found.

Example 3. Consider the AF $F = (\{t, a\}, \{\})$ and let us suppose we have an agent \mathcal{A} who has the goal to defeat argument t and can potentially advance the arguments b and c , whereby b will attack t and be attacked by a , while c will attack a and be attacked by b . This means our agent can construct the following AFs (or abstain from any changes and keep F):

- $F_b = (\{t, a, b\}, \{(a, b), (b, t)\})$;
- $F_c = (\{t, a, c\}, \{(c, a)\})$;
- $F_{b,c} = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$.

Figure 2 depicts the graphs of the argumentation frameworks. Clearly, F and F_b yield the extension $\{a, t\}$, whereas F_c yields the extension $\{c, t\}$. Advancing a consistent subset of the arguments $\{b, c\}$ does not allow \mathcal{A} to reach the goal of rejecting t . However, assuming we use the classical argumentation semantics that Dung has defined in his seminal paper that introduces abstract argumentation [19], advancing the inconsistent arguments c and a leads to the inference of the empty set from $F_{b,c}$, i.e., advancing “kettle logic” arguments is the only way \mathcal{A} can achieve their objective.

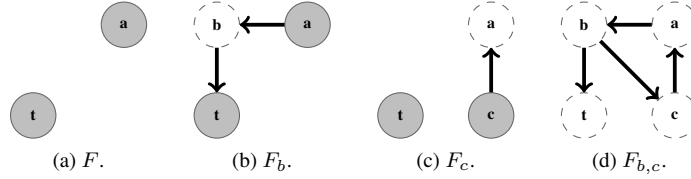


Figure 2: Kettle logic: only by advancing the inconsistent set of arguments $\{a, c\}$ can our agent achieve the rejection of the target argument t . Here and henceforth, arguments that are in all extensions that the applied semantics yield(s) have a grey background and solid border; arguments that are in at least one extension have a solid border (and white background); arguments that are in no extension have a dashed border (and white background).

Let us claim here that while in the previous example, the application of some other argumentation semantics does not facilitate the use of kettle logic, generally, all “reasonable”³ argumentation semantics are affected by kettle logic. The same applies to three-valued labelling-based semantics that given an argumentation framework and an extension that has been inferred from it, define all arguments that are not in the extension and also not attacked by arguments in the extension as *undecided* [13, 14, 35].

³We use this term to exclude naive semantics (to be defined later), which can be considered a helper semantics that is extended by more useful semantics.

Given such semantics, kettle logic still succeeds in our example to change the status of the target argument from clearly accepted/inferred to undecided⁴.

In this paper, we provide a comprehensive formal analysis that uses the intuition provided by the previous example as a starting point, leading to the following results.

1. We formalise different forms of kettle logic for abstract argumentation, depending on the effect the advancement of inconsistent arguments has, *i.e.*, depending on whether kettle logic leads to the clear rejection of an argument that has previously been unambiguously inferred (rejection kettle logic), to the status change from an unambiguously inferred argument to a credulously (ambiguously) inferred argument (credulous kettle logic), or to the rejection of an argument that has been credulously inferred (credulous rejection kettle logic). These results are presented in Section 3.
2. A formal analysis shows that all of the surveyed “reasonable” argumentation semantics of the three well-established semantics families – admissible set-based, weak admissible set-based, and naive set-based semantics – are vulnerable to all forms of kettle logic. In the cases of admissible set-based and weak admissible set-based semantics, the corresponding propositions take a principle-based approach, stating that, given the intuitive assumption that the semantics yields at least one extension for some rather simple (specific) AFs, (weak) admissibility and weak reinstatement (a very basic, intuitive principle) are sufficient to prove the violation of all forms of kettle logic (Section 4). Note that we provide a brief informal discussion of the kettle logic fallacy considering additional argument acceptance statuses in Section 6.
3. We demonstrate how kettle logic can be mitigated in some cases, either by constraining the selection of extensions that an argumentation semantics yield, or by limiting the advancement of arguments by an agent by applying specific structural constraints (Section 5).

2 Formal Preliminaries

This section introduces the formal preliminaries of the work that this paper presents. An Argumentation Framework (AF) is a tuple (AR, AT) where AR is a set of arguments and $AT \subseteq AR \times AR$. Given an AF $F = (AR, AT)$, $a, b \in AR$, and $S, S' \subseteq AR$ we say that: “ a attacks b ” (in F) iff $(a, b) \in AT$; “ S attacks b ” iff $\exists c \in S$ s.t. c attacks b ; “ a attacks S ” iff $\exists d \in S$ s.t. a attacks d ; “ S attacks S' ” iff $\exists f \in S$ s.t. f attacks S' ; “ S defends a ” iff $\forall g \in AR$ s.t. g attacks a it holds that S attacks g ; “ a is self-attacking” iff $(a, a) \in AT$; “ S is conflict-free” iff $\forall x \in S$ it does not hold that S attacks x ; “ a is unattacked” iff AR does not attack a ; “ S is unattacked” iff AR does not attack S . We denote $\{a \mid a \in AR, S \text{ attacks } a\}$ by S^+ . Also, we call $(S, (S \times S) \cap AT)$ the “restriction of F to S ” (denoted by $F \downarrow_S$) and we say that F is “without odd cycles”

⁴For the sake of conciseness, we do not formally analyse three-valued labelling semantics in this paper. However, we provide a brief discussion in Section 6.

iff $\forall y \in AR$ there exists no directed path⁵ of odd-length from y to y ; we denote the Strongly Connected Components (SCCs) of F by $SCCS(F)$. We denote the class of all AFs by \mathcal{FF} , the class of all AFs whose arguments are not self-attacking by \mathcal{FS} , the class of all AFs that are without odd cycles by \mathcal{FO} , and the class of argumentation frameworks with finite sets of arguments that are acyclic directed graphs by \mathcal{FN} .

An argumentation semantics is a function σ that, given an argumentation framework (AR, AT) , returns a set of *extensions* ES s.t. $ES \subseteq 2^{AR}$. The classical semantics presented in Dung’s seminal paper that introduces abstract argumentation are based on the notion of an admissible set [19]. Given an AF (AR, AT) , a set $S \subseteq AR$ is admissible iff S is conflict-free and $\forall a \in S$ it holds that S defends a ; $ad(F)$ denotes the set of all $S' \subseteq AR$ that are admissible (in F). Below are the definitions of the classical semantics.

Definition 1 (Admissible Set-based Semantics [19]). *Let $F = (AR, AT)$ be an argumentation framework. An admissible set $S \subseteq AR$ is a:*

- stable extension of F iff S attacks every argument that does not belong to S . $\sigma_{st}(F)$ denotes all stable extensions of F ;
- preferred extension of F iff S is a maximal (w.r.t. set inclusion) admissible subset of AR . $\sigma_{pr}(F)$ denotes all preferred extensions of F ;
- complete extension of F iff every argument that is defended by S belongs to S . $\sigma_{co}(F)$ denotes all complete extensions of F ;
- grounded extension of F iff S is the minimal (w.r.t. set inclusion) complete extension of F . $\sigma_{gr}(F)$ denotes all grounded extensions of F .

More recently, weak admissible set-based argumentation semantics have been introduced by Baumann *et al.* [8]⁶. Roughly speaking, these semantics relax the admissibility constraint of the classical semantics as introduced by Dung: if an argument indirectly attacks itself (through an odd cycle that is not broken by “successful” attacks by arguments whose acceptance considering weak admissibility has already been established), the argument can be considered invalid and an extension does not need to defend itself against it. Let us first provide the definitions of weak admissibility and its preliminaries that formalise this intuition.

Definition 2 (*E*-Reduct, Weak Admissibility and Weak Defence [8]). *Let $F = (AR, AT)$ be an argumentation framework and let $E, S \subseteq AR$.*

- The *E*-reduct of F is the argumentation framework $F^E = F \downarrow_{E^*}$, where $E^* = AR \setminus (E \cup E^+)$.

⁵Note that an AF is a directed graph and hence we can make use of the following well-known notions given an AF $F = (AR, AT)$: i) given $a, b \in AR$, there exists a directed path from a to b (in F), denoted by $P_{F,a,b}$, iff $\exists a_1, \dots, a_n \in AR$ such that $a_1 = a, a_n = b$ and for $1 \leq i < n$ it holds that $(a_i, a_{i+1}) \in AT$; ii) F is acyclic iff $\nexists a \in AR$ such that $P_{F,a,a}$ holds true; iii) $S \subseteq AR$ is a strongly connected component (of F) iff $\forall a, b \in S, P_{F,a,b}$ and $P_{F,b,a}$ hold true and $\nexists c \in AR \setminus S$ such that $P_{F,a,c}$ and $P_{F,c,a}$ hold true.

⁶Note that the underlying idea was independently discovered several years earlier by Kakas and Mancarella [26].

- E is weakly admissible in F , denoted by $E \in ad^w(F)$, iff E is conflict-free and $\forall a \in AR$ s.t. a attacks E it holds true that $a \notin \bigcup_{A \in ad^w(F^E)} A$.
- E weakly defends S iff $\forall a \in AR$ s.t. a attacks S the following statement holds true:

$$E \text{ attacks } a \text{ or} \\ (a \notin \bigcup_{A \in ad^w(F^E)} A, a \notin E \text{ and } S \subseteq S' \in ad^w(F))$$

Now, we can provide the definitions of the weakly admissible counterparts of preferred, complete, and grounded semantics.

Definition 3 (Weak Admissible Set-based Semantics [8]). *Let $F = (AR, AT)$ be an argumentation framework. $E \subseteq AR$ is a:*

- weakly preferred extension of F iff E is \subseteq -maximal in $ad^w(F)$. $\sigma_{wpr}(F)$ denotes all weakly preferred extensions of F .
- weakly complete extension of F iff $E \in ad^w(F)$ and for any set S , such that $E \subseteq S$ and S is weakly defended by E , it holds true that $S \subseteq E$. $\sigma_{wco}(F)$ denotes all weakly complete extensions of F .
- weakly grounded extension of F iff E is \subseteq -minimal in $\sigma_{wco}(F)$. $\sigma_{wgr}(F)$ denotes all weakly grounded extensions of F .

Yet another family of argumentation semantics is based on the notion of \subseteq -maximal conflict-free sets of arguments, so-called *naive* sets.

Definition 4 (Naive and Stage Semantics [33]). *Let $F = (AR, AT)$ be an argumentation framework and let $S \subseteq AR$.*

- S is a naive extension of F iff S is a maximal conflict-free subset of AR w.r.t. set inclusion. $\sigma_{na}(F)$ denotes all naive extensions of F .
- S is a stage extension of F iff S is conflict-free and $S \cup S^+$ is maximal w.r.t. set inclusion, i.e., $\nexists S' \subseteq AR$, such that S' is a conflict-free set and $S \cup S^+ \subset S' \cup S'^+$. $\sigma_{stg}(F)$ denotes all stage extensions of F .

Additional naive set-based argumentation semantics rely on recursing the SCCs of an argumentation framework. For this approach, the UP function is a prerequisite.

Definition 5 (UP Function [4]). *Let $F = (AR, AT)$ be an argumentation framework and let $E \subseteq AR$, $S \subseteq AR$. We define $UP_F(S, E) = S \setminus (E \setminus S)^+$.*

CF2 and stage2 semantics recurse the SCCs of an argumentation framework starting with “top-level” (unattacked) SCCs, applying naive and stage semantics, respectively, on SCC-level, and the UP function to determine the effect of successful attacks from one SCC to another.

Definition 6 (CF2 and Stage2 Semantics [4, 21]). *Let $F = (AR, AT)$ be an argumentation framework and let $E \subseteq AR$. E is a CF2 extension of F iff:*

- E is a naive extension of F if $|SCCS(F)| = 1$;
- $\forall S \in SCCS(F)$, $(E \cap S)$ is a CF2 extension of $F \downarrow_{UP_F(S,E)}$, otherwise.

$\sigma_{CF2}(F)$ denotes all CF2 extensions of F .

E is a stage2 extension of F iff:

- E is a stage extension of F if $|SCCS(F)| = 1$;
- $\forall S \in SCCS(F)$, $(E \cap S)$ is a stage2 extension of $F \downarrow_{UP_F(S,E)}$, otherwise.

$\sigma_{stg2}(F)$ denotes all stage2 extensions of F .

A detailed comparison between argumentation semantics is beyond the scope of the paper. For this, we point the reader to dedicated works [2, 8]. Still, let us introduce examples that highlight the differences between the three semantics families (admissible set-based, weak admissible set-based, and naive set-based) and that provide a rough explanation of how SCC-recursive semantics work.

Example 4. *For the difference between admissible set-based, weak admissible set-based, and naive set-based semantics, consider $F = (\{a, b, c, d\}, \{(a, b), (b, c), (c, a), (c, d)\})$ (Figure 3a). Because of the three-cycle “ a attacks b attacks c attacks a ”, no argument is admissible in F ; $\sigma_{st}(F) = \emptyset$ and $\sigma_{pr}(F) = \sigma_{co}(F) = \sigma_{gr}(F) = \{\emptyset\}$. In contrast, weak admissible set-based semantics consider the three-cycle self-defeating (roughly speaking): neither $\{a\}$, nor $\{b\}$, nor $\{c\}$ are weakly admissible and hence, $\{d\}$ does not need to defend itself against c . Consequently, we have $\sigma_{wpr}(F) = \sigma_{wc}(F) = \sigma_{wgr}(F) = \{\{d\}\}$. Yet differently, naive set-based semantics assume that we should be able to infer either a or b or c from the three-cycle, which then yields (considering the conflict-freeness and maximality constraints) $\sigma_{na}(F) = \sigma_{stg}(F) = \sigma_{CF2}(F) = \sigma_{stg2}(F) = \{\{a, d\}, \{b, d\}, \{c\}\}$.*

To highlight how SCC-recursive CF2 and stage2 semantics work, consider $F' = (\{a, b, c, d, e\}, \{(a, b), (a, c), (a, d), (b, c), (b, d), (c, a), (c, d), (d, e)\})$ (Figure 3b). Intuitively, we consider the three-cycle, which is the only unattacked strongly connected component in F' , i.e., we first look at $F^ = F' \downarrow_{\{a,b,c\}}$. In the case of CF2 semantics, we have $\sigma_{na}(F^*) = \{\{a\}, \{b\}, \{c\}\}$; given stage2 semantics, we have $\sigma_{stg}(F^*) = \{\{a\}\}$. Because every extension in either $\sigma_{na}(F^*)$ or $\sigma_{stg}(F^*)$ successfully attacks the only argument d in the “next” SCC, d is discarded, and hence e – the only argument in the following SCC – is obviously included in all extensions. Hence, we have $\sigma_{CF2}(F^*) = \{\{a, e\}, \{b, e\}, \{c, e\}\}$ and $\sigma_{stg2}(F^*) = \{\{a, e\}\}$.*

Given an argumentation semantics that may yield more than one extension for a given AF, we can distinguish between skeptically accepted arguments, which are contained by *every* of the extensions, and credulously accepted arguments, which are contained by *any* (at least one) of the extensions.

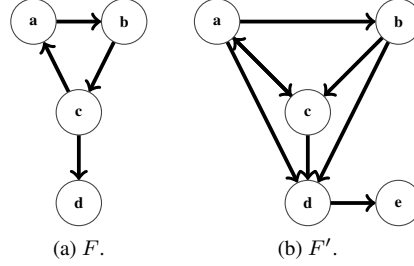


Figure 3: F illustrates the difference between admissible set-based, weak admissible set-based, and naive set-based semantics; F' the difference between CF2 and stage2 semantics.

Definition 7 (Skeptical and Credulous Acceptance). *Let σ be an argumentation semantics, let $F = (AR, AT)$ be an AF, and let $a \in AR$. We say that a is skeptically accepted in F (w.r.t. σ), denoted by $a \in \sigma^\cap(F)$, iff $a \in \bigcap_{E \in \sigma(F)} E$; we say that a is credulously accepted in F (w.r.t. σ), denoted by $a \in \sigma^\cup(F)$, iff $a \in \bigcup_{E \in \sigma(F)} E$.*

Argumentation principles have been defined to facilitate the formal analysis of argumentation semantics [3, 32]. Some well-known principles are relevant to our analysis of Kettle logic. Intuitively, the three principles of *admissibility*, *weak admissibility*, and *naivety*, which are central to the three semantics families, are important; in addition, we need the principle of weak reinstatement (roughly: “arguments whose defence can be traced back to unattacked arguments should always be inferred”). As a prerequisite for the latter principle, we define the notions of strong admissibility and strong defence.

Definition 8 (Strong Admissibility and Strong Defence [32]). *Given an AF $F = (AR, AT)$, we say that an argument $a \in AR$ is strongly defended (in F) by a set $S \subseteq AR$ iff $\forall b \in AR$ s.t. b attacks a it holds that $\exists c \in S$ s.t. c attacks b and c is strongly defended by $S \setminus \{a\}$. S is strongly admissible (in F) iff it is conflict-free and $\forall d \in S$, S strongly defends d .*

Now, we can provide the definitions of the principles.

Definition 9 (Argumentation Principles [3, 32]). *An argumentation semantics σ satisfies the:*

Admissibility principle iff $\forall F \in \mathcal{FF}$ it holds that $\sigma(F) \subseteq ad(F)$;

Weak admissibility principle iff $\forall F \in \mathcal{FF}$ it holds that $\sigma(F) \subseteq ad^w(F)$;

Naivety principle iff $\forall F \in \mathcal{FF}$ it holds that $\sigma(F) \subseteq \sigma_{na}(F)$;

Weak reinstatement principle iff $\forall F \in \mathcal{FF}$, $\forall E \in \sigma(F)$, $\forall a \in AR$ it holds that iff E strongly defends a then $a \in E$.

Finally, let us provide the definition of argumentation framework *expansions*, *normal expansions*, and *expansion chains* that can be used to model argumentation dynamics.

Definition 10 (Expansions, Normal Expansions, and Expansion Chains [6]). *Let $F = (AR, AT)$ and $F' = (AR', AT')$ be argumentation frameworks.*

- F' is an expansion of F , denoted by $F \preceq F'$, iff $AR \subseteq AR'$ and $AT \subseteq AT'$.
- F' is a normal expansion of F , denoted by $F \preceq_N F'$, iff $F \preceq F'$ and $(AT' \setminus AT) \cap (AR \times AR) = \emptyset$.

A sequence of argumentation frameworks $\langle F_0, \dots, F_n \rangle$ is an expansion chain iff for $0 \leq i < n$ it holds true that $F_i \preceq_N F_{i+1}$.

Intuitively, an expansion adds arguments and attacks to an argumentation framework, a normal expansion is an expansion that does not add attacks between previously existing arguments, and an expansion chain is a sequence of normal expansions. Considering the argumentation frameworks in Figure 2, we can observe that the sequences of argumentation frameworks $\langle F, F_b, F_{b,c} \rangle$ and $\langle F, F_c, F_{b,c} \rangle$ are expansion chains, because $F \preceq_N F_b$, $F \preceq_N F_c$, $F_b \preceq_N F_{b,c}$, and $F_c \preceq_N F_{b,c}$ hold. In contrast, $\langle F, F_b, F_c, F_{b,c} \rangle$ is not an expansion chain, because $F_b \preceq_N F_c$ does not hold ($F_b \preceq F_c$ does not hold, either).

3 Defining Kettle Logic

We intuitively have three different forms of kettle logic⁷, given an AF $F = (AR, AT)$, a semantics σ , a target argument $t \in AR$, and a set of inconsistent arguments S that normally expand F (adding new arguments to AR and attacks to AT , with no new attack being in $AR \times AR$):

Rejection kettle logic. Our target argument is skeptically accepted in F and the advancement of S expands F to F' such that t is not credulously accepted in F' . No conflict-free subset of S exists that allows for a normal expansion to an AF F'' such that t is not credulously accepted in F'' .

Credulous kettle logic. Our target argument is skeptically accepted in F and the advancement of S expands F to F' such that t is not skeptically accepted in F' . No conflict-free subset of S exists that allows for a normal expansion to an AF F'' such that t is not skeptically accepted in F'' .

Credulous rejection kettle logic. Our target argument is credulously accepted in F and the advancement of S expands F to F' such that t is not credulously accepted in F' . No conflict-free subset of S exists that allows for a normal expansion to an AF F'' such that t is not credulously accepted in F'' .

The formal definitions of these forms of kettle logic follow below.

Definition 11 (Kettle Logic). *We say that, with respect to a class \mathcal{F} of argumentation frameworks, an argumentation semantics σ is vulnerable to:*

⁷Given an argumentation semantics that is not universally defined, *i.e.*, that, for every argumentation framework, yields at least one extension, one could define one or several additional variants of kettle logic that explicitly cover cases where no extensions can be inferred from an argumentation framework.

rejection kettle logic *iff there exists an AF $F \in \mathcal{F}$, $F = (AR, AT)$, an argument $t \in AR$, and a set of arguments $S \subseteq AR$ s.t. S is not conflict-free and the following statements hold true:*

- $t \in \sigma^\cap(F \downarrow_{AR \setminus S})$;
- $t \notin \sigma^\cup(F)$;
- $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cup(F \downarrow_{(AR \setminus S) \cup S'})$.

credulous kettle logic *iff there exists an AF $F \in \mathcal{F}$, $F = (AR, AT)$, an argument $t \in AR$, and a set of arguments $S \subseteq AR$ s.t. S is not conflict-free and the following statements hold true:*

- $t \in \sigma^\cap(F \downarrow_{AR \setminus S})$;
- $t \notin \sigma^\cap(F)$;
- $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cap(F \downarrow_{(AR \setminus S) \cup S'})$.

credulous rejection kettle logic *iff there exists an AF $F \in \mathcal{F}$, $F = (AR, AT)$, an argument $t \in AR$, and a set of arguments $S \subseteq AR$ s.t. S is not conflict-free and the following statements hold true:*

- $t \in \sigma^\cup(F \downarrow_{AR \setminus S})$;
- $t \notin \sigma^\cup(F)$;
- $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cup(F \downarrow_{(AR \setminus S) \cup S'})$.

If we do not specify the class of argumentation frameworks in our analysis of kettle logic, it means we refer to the class \mathcal{FF} of all AFs. We say that an argumentation semantics σ is vulnerable to all forms of kettle logic iff σ is vulnerable to rejection kettle logic, as well as to credulous kettle logic and to credulous rejection kettle logic. Conversely, we say that σ is not vulnerable to any form of kettle logic iff σ is not vulnerable to rejection kettle logic, and neither to credulous kettle logic, nor to credulous rejection kettle logic. If and only if an argumentation semantics is not vulnerable to (rejection, credulous, or credulous rejection) kettle logic then we can say that the semantics is *resistant* to (rejection, credulous, or credulous rejection) kettle logic. The reader may have noted that we informally describe kettle logic using normal expansions although we do not make use of this notion in the formal definition. For an explicit formal characterisation of kettle logic using normal expansions, we point to Section 5.

We observe that if an argumentation semantics is vulnerable to rejection kettle logic, it is vulnerable to credulous rejection kettle logic.

Proposition 1. *If an argumentation semantics σ is vulnerable to rejection kettle logic, it is also vulnerable to credulous rejection kettle logic.*

Note that all proofs are available in the appendix.

4 Kettle Logic in Abstract Argumentation Semantics

We can show that the semantics defined in Section 2, except for naive semantics, are vulnerable to all forms of kettle logic. In contrast, naive semantics is not vulnerable to any form of kettle logic. To cover Dung’s classical semantics, let us start by introducing a principle-based observation, based on the example provided in the introduction (Example 3). Note that for Proposition 2, we make the additional (intuitive, yet important) assumption that the semantics yields at least one extension for the AFs F , F_b , and F_c in our counterexample (Figure 2); *e.g.*, we can say that we assume our semantics is universally defined for \mathcal{FN} .

Proposition 2. *Every argumentation semantics σ that satisfies admissibility and weak reinstatement is vulnerable to all forms of kettle logic.*

From this observation, it follows straight-forwardly that stable, preferred, complete, and grounded semantics are vulnerable to all forms of kettle logic.

Corollary 1. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr\}$. σ_x is vulnerable to all forms of kettle logic.*

Example 3 also shows that kettle logic vulnerability is not limited to argumentation frameworks with self-attacking arguments.

Corollary 2. *Every argumentation semantics σ that satisfies admissibility and weak reinstatement is vulnerable to all forms of kettle logic w.r.t. \mathcal{FS} .*

This result then applies to the classical Dung semantics as well, because all of them satisfy admissibility and weak reinstatement [32].

Corollary 3. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr\}$. σ_x is vulnerable to all forms of kettle logic w.r.t. \mathcal{FS} .*

Analogously, we can show that weak admissible set-based semantics that satisfy weak reinstatement are vulnerable to all forms of kettle logic (see Figure 4 for the counterexample⁸). Note that for Proposition 3, we make the additional (intuitive, yet important) assumption that the semantics yields at least one extension for the AFs F^* , F_c^* , and F_d^* in our counterexample (Figure 4); *e.g.*, we can say that we assume our semantics is universally defined for finite AFs in \mathcal{FF} .

Proposition 3. *Every argumentation semantics σ that satisfies weak admissibility and weak reinstatement is vulnerable to all forms of kettle logic.*

From this, it follows that weakly preferred, weakly complete, and weakly grounded semantics are vulnerable to all forms of kettle logic, because these semantics satisfy weak admissibility (obviously, by definition) and weak reinstatement [8].

Corollary 4. *Let σ_x be an argumentation semantics s.t. $x \in \{wpr, wco, wgr\}$. σ_x is vulnerable to all forms of kettle logic.*

Again, the observation is not limited to argumentation frameworks with self-attacking arguments.

⁸Still, the proof is provided more formally in the appendix.

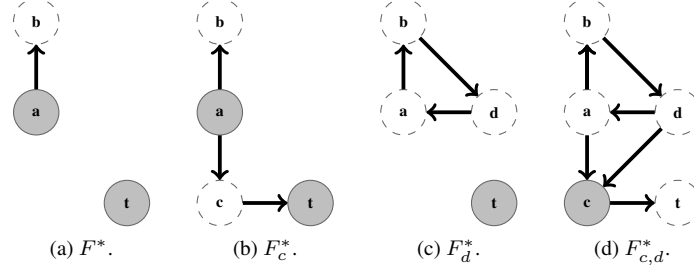


Figure 4: Kettle logic in weakly admissible set-based semantics.

Corollary 5. *Let σ_x be an argumentation semantics s.t. $x \in \{wpr, wco, wgr\}$. σ_x is vulnerable to all forms of kettle logic w.r.t. \mathcal{FS} .*

In contrast, naive semantics is not vulnerable to any form of kettle logic.

Proposition 4. *σ_{na} is not vulnerable to any form of kettle logic.*

However, the other naive set-based semantics – stage, CF2, and stage2 semantics – are vulnerable to kettle logic. Using an example (Figure 2), we can show that the semantics are vulnerable to credulous kettle logic.

Proposition 5. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic.*

Again, the vulnerability is not limited to argumentation frameworks with self-attacking arguments.

Corollary 6. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic w.r.t. \mathcal{FS} .*

Using another example (Figure 5), we can show that stage, CF2, and stage2 semantics are vulnerable to rejection kettle logic and credulous rejection kettle logic.

Proposition 6. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic.*

The example depicted in Figure 5 also shows that all of the surveyed semantics except for (weakly) grounded⁹ and naive semantics are vulnerable to rejection kettle logic and credulous rejection kettle logic with respect to \mathcal{FO} .

Proposition 7. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, wpr, wco, stg, CF2, stg2\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic w.r.t. \mathcal{FO} .*

⁹Note that grounded and weakly grounded semantics are vulnerable to rejection kettle logic and credulous rejection kettle logic with respect to \mathcal{FO} ; however, a different example is required to demonstrate this.

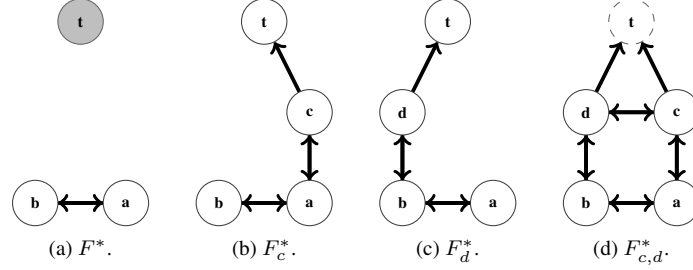


Figure 5: Rejection kettle logic and credulous rejection kettle logic in stage, CF2, and stage2 semantics.

An additional example (Figure 6) allows us to show that grounded and weakly grounded semantics are vulnerable to rejection kettle logic and credulous rejection kettle logic with respect to \mathcal{FO} as well.

Proposition 8. *Let σ_x be an argumentation semantics s.t. $x \in \{wgr, gr\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic w.r.t. \mathcal{FO} .*

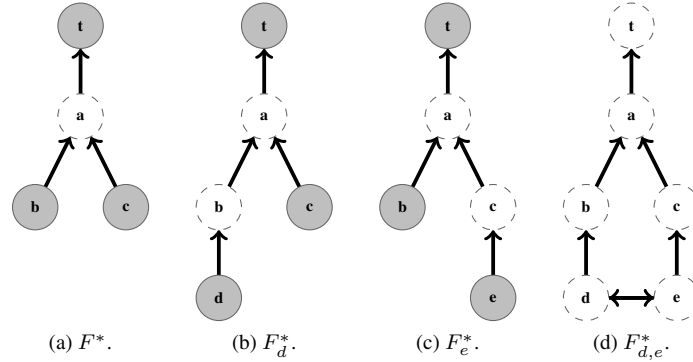


Figure 6: Rejection kettle logic, credulous kettle logic, and credulous rejection kettle logic in (weakly) grounded semantics given an argumentation framework without odd cycles.

Using the example in Figure 6, we can also show that all of the surveyed semantics except for naive semantics are vulnerable to credulous kettle logic with respect to \mathcal{FO} .

Proposition 9. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic w.r.t. \mathcal{FO} .*

Before we conclude the analysis section, let us remark that our definition of kettle logic focuses on scenarios where the acceptance status of the target argument changes

from “more accepted“ to “less accepted”, colloquially speaking. However, if we were to define forms of kettle logic in which the status of the target argument changes from “less accepted“ to “more accepted”, a rather straightforward change to the counterexamples would produce analogous negative results: we merely needed to normally expand the AFs in the example by “adding” a new argument t' such that t' is only attacked by t (and does not attack any other argument) and then make t' our new target argument. The reader may verify that then, given all of the surveyed argumentation semantics with the exception of naive semantics, t' is sceptically accepted in a given AF iff t is neither sceptically nor credulously accepted; t' is not sceptically accepted iff t is credulously accepted; t' is neither sceptically nor credulously accepted iff t is sceptically accepted. Let us informally claim that the positive result in the case of naive semantics would remain as well, roughly because normally expanding an AF in which an argument is credulously accepted cannot make this argument sceptically accepted in the normal expansion (and only self-attacking arguments are neither sceptically nor credulously accepted in any AF given naive semantics). Hence, we claim that while a rigid formalisation and analysis of these potentially additional forms of kettle logic may be to some extent technically interesting, it would not fundamentally affect the relevance of the results presented in this paper.

5 Mitigating Kettle Logic

In the previous section, we have shown that all argumentation semantics, with the exception of naive semantics, are vulnerable to all forms of kettle logic and that vulnerability to kettle logic is neither limited to argumentation frameworks with self-attacking arguments nor to argumentation frameworks with odd cycles. Given these *negative* results, an obvious question is whether one can devise approaches that avoid the occurrence of kettle logic or mitigate its consequences. As a prerequisite for the approaches, we formally relate kettle logic to expansion chains. Then, we present three of such approaches and discuss their advantages and shortcomings.

5.1 Kettle Logic and Argumentation Dialogues as Expansion Chains

In our approach to defining kettle logic, we assume an *argumentation dialogue* in which we start with an argumentation framework that is then *normally expanded* one or several times by one or several agents. To make this idea explicit, let us define *chain kettle logic* (reflecting the notion of an expansion chain) using normal expansions.

Definition 12 (Chain Kettle Logic). *We say that, with respect to a class of argumentation frameworks \mathcal{F} , an argumentation semantics is vulnerable to:*

chain rejection kettle logic *iff there exist AFs $F, F' \in \mathcal{F}$, $F = (AR, AT)$, $F' = (AR', AT')$ s.t. $F' \preceq_N F$, s.t. $AR \setminus AR'$ is not conflict-free, and $t \in AR'$ and the following statements hold true:*

- $t \in \sigma^\cap(F')$;
- $t \notin \sigma^\cup(F)$;

- $\nexists F'' \in \mathcal{F}, F'' = (AR'', AT'')$ s.t. $F' \preceq_N F'', F'' \preceq_N F, AR'' \setminus AR'$ is conflict-free, and $t \notin \sigma^\cup(F'')$.

chain credulous kettle logic iff there exist AFs $F, F' \in \mathcal{F}, F = (AR, AT), F' = (AR', AT')$ s.t. $F' \preceq_N F$ s.t. $AR \setminus AR'$ is not conflict-free, and $t \in AR'$ and the following statements hold true:

- $t \in \sigma^\cap(F')$;
- $t \notin \sigma^\cap(F)$;
- $\nexists F'' \in \mathcal{F}, F'' = (AR'', AT'')$ s.t. $F' \preceq_N F'', F'' \preceq_N F, AR'' \setminus AR'$ is conflict-free, and $t \notin \sigma^\cap(F'')$.

chain credulous rejection kettle logic iff there exist AFs $F, F' \in \mathcal{F}, F = (AR, AT), F' = (AR', AT')$ s.t. $F' \preceq_N F$ s.t. $AR \setminus AR'$ is not conflict-free, and $t \in AR'$ and the following statements hold true:

- $t \in \sigma^\cup(F')$;
- $t \notin \sigma^\cup(F)$;
- $\nexists F'' \in \mathcal{F}, F'' = (AR'', AT'')$ s.t. $F' \preceq_N F'', F'' \preceq_N F, AR'' \setminus AR'$ is conflict-free, and $t \notin \sigma^\cup(F'')$.

Analogously to kettle logic, if we do not specify the class of argumentation frameworks in our analysis of chain kettle logic, it means we refer to the class \mathcal{FF} of all AFs. Intuitively, kettle logic and chain kettle logic are equivalent.

Lemma 1. *An argumentation semantics σ is:*

1. *vulnerable to chain rejection kettle logic iff it is vulnerable to rejection kettle logic;*
2. *vulnerable to chain credulous kettle logic iff it is vulnerable to credulous kettle logic;*
3. *vulnerable to chain credulous rejection kettle logic iff it is vulnerable to credulous rejection kettle logic.*

Let us define *expansion chain dialogues*, which we can then use as a formal framework for managing kettle logic.

Definition 13 (Expansion Chain Dialogue). *An expansion chain dialogue is a quintuple $(\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$ where \mathcal{AS} is a set of agents, $\langle F_0, \dots, F_n \rangle$ is an expansion chain, and $\langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle$ is a sequence of agents, i.e., for $0 \leq i \leq n$ it holds that $\mathcal{A}_i \in \mathcal{AS}$; given $F_0 = (AR_0, AT_0)$ it must hold that $t \in AR_0$; we call t the “target argument” or the “topic argument” of the dialogue; σ is an argumentation semantics; for $\mathcal{A}_i, F_i = (AR_i, AT_i), F_{i-1} = (AR_{i-1}, AT_{i-1})$, we say that “ \mathcal{A} ’s utterance at i is $(AR_i \setminus AR_{i-1}, AT_i \setminus AT_{i-1})$ ”¹⁰ if $i > 0$ and “ \mathcal{A} ’s utterance at i is F_i ”, otherwise.*

¹⁰Here, $(AR_i \setminus AR_{i-1}, AT_i \setminus AT_{i-1})$ is an (arguments, attacks)-tuple, but not necessarily an argumentation framework.

Now, we can define kettle logic in expansion chain dialogues.

Definition 14 (Kettle Logic in Expansion Chain Dialogues). *Let $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$ be an expansion chain dialogue. A kettle logic fallacy occurs in D iff there exist AFs F_0, F_i , $0 < i \leq n$, $F_0 = (AR_0, AT_0)$, $F_i = (AR_i, AT_i)$ s.t. $AR_i \setminus AR_0$ is not conflict-free and the following statements hold true:*

rejection kettle logic:

- $t \in \sigma^\cap(F_0)$;
- $t \notin \sigma^\cup(F_i)$;
- $\nexists F' \in \mathcal{F}$, $F' = (AR', AT')$ s.t. $F_0 \preceq_N F'$, $F' \preceq_N F_i$, $AR' \setminus AR_0$ is conflict-free, and $t \notin \sigma^\cup(F')$.

credulous kettle logic:

- $t \in \sigma^\cap(F_0)$;
- $t \notin \sigma^\cap(F_i)$;
- $\nexists F' \in \mathcal{F}$, $F' = (AR', AT')$ s.t. $F_0 \preceq_N F'$, $F' \preceq_N F_i$, $AR' \setminus AR_0$ is conflict-free, and $t \notin \sigma^\cap(F')$.

credulous rejection kettle logic:

- $t \in \sigma^\cup(F_0)$;
- $t \notin \sigma^\cup(F_i)$;
- $\nexists F' \in \mathcal{F}$, $F' = (AR', AT')$ s.t. $F_0 \preceq_N F'$, $F' \preceq_N F_i$, $AR' \setminus AR_0$ is conflict-free, and $t \notin \sigma^\cup(F')$.

Obviously, a kettle logic fallacy can only occur in an expansion chain dialogue if the semantics that is used in the dialogue is vulnerable to kettle logic.

Corollary 7. *If a rejection/credulous/credulous rejection kettle logic fallacy occurs in an expansion chain dialogue $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$ then σ is vulnerable to rejection/credulous/credulous rejection kettle logic.*

Also, if a rejection kettle logic fallacy occurs in an expansion chain dialogue then a credulous rejection kettle logic fallacy occurs in the dialogue as well.

Proposition 10. *If a rejection kettle logic fallacy occurs in an expansion chain dialogue D , then a credulous rejection kettle logic fallacy occurs in D .*

Note that while the proof is somewhat analogous to the proof of Proposition 1, the propositions are technically different. Let us go back to our motivating examples to explain the notion of an expansion chain dialogue and to demonstrate how we can identify kettle logic fallacies in such a dialogue and how the utterances of several agents may lead to kettle logic.

Example 5 (Examples 1 and 3 continued). Consider Example 1. We can model this classical kettle logic example (which is not considered a case of kettle logic from our abstract argumentation-based perspective) as an expansion chain dialogue, in particular as $D = (\{\mathcal{A}_0, \mathcal{A}_1\}, \langle F_0, F_1 \rangle, \langle \mathcal{A}_0, \mathcal{A}_1 \rangle, t, \sigma_{pr})$, where $F_0 = (\{t\}, \{\})$ and $F_1 = (\{t, a, b, c\}, \{(a, t), (a, b), (a, c), (b, t), (b, a), (b, c), (c, t), (c, a), (c, b)\})$ (see Figure 1; $F_1 = F$). In the dialogue we have two utterances: first \mathcal{A}_0 utters $(\{t\}, \{\})$ and subsequently, \mathcal{A}_1 utters $(\{a, b, c\}, \{(a, t), (a, b), (a, c), (b, t), (b, a), (b, c), (c, t), (c, a), (c, b)\})$. What we see is that no kettle logic fallacy occurs in D ! Let us go through the details for rejection kettle logic. Although $t \in \sigma_{pr}^\cap(F_0)$ and $t \notin \sigma_{pr}^\cap(F_1)$, it is clear that for \mathcal{A}_1 , it would be sufficient to utter $(\{a\}, \{(a, t)\})$ – note that $\{a\}$ is clearly conflict-free – which leads to the argumentation framework $F'_1 = (\{t, a\}, \{(a, t)\})$; obviously, $t \notin \sigma_{pr}^\cap(F'_1)$.

Now, consider Example 3 (Figure 2). Here, we have the expansion chain dialogue $D' = (\{\mathcal{A}_0, \mathcal{A}_1\}, \langle F_0, F_1 \rangle, t, \sigma_{pr})$, where $F_0 = (\{t, a\}, \{\})$ and $F_1 = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$ (see Figure 2; $F_0 = F$ and $F_1 = F_{b,c}$). First, \mathcal{A}_0 utters $(\{t, a\}, \{\})$ and then \mathcal{A}_1 utters $(\{b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$. We can see all kettle logic fallacies occur in D' . Consider, for example, rejection kettle logic: $t \in \sigma_{pr}^\cap(F_0)$ and $t \notin \sigma_{pr}^\cap(F_1)$; $\{b, c\}$ is not conflict-free and for $F'_1 = F_1 \downarrow_{\{t, a, b\}}$ and $F''_1 = F_1 \downarrow_{\{t, a, c\}}$ it holds that $t \in \sigma_{pr}^\cap(F'_1)$ and $t \in \sigma_{pr}^\cap(F''_1)$.

Given the notion of kettle logic in expansion chain dialogues, we can now introduce and discuss kettle logic mitigation approaches.

5.2 Approach 1: Prohibiting the Advancement of Kettle Logic Arguments

Given our definition of kettle logic in expansion chain dialogues (Definition 14), we can define which utterances are *kettle logic-legal*, which enables us to straightforwardly prohibit utterances that lead to kettle logic fallacies.

Definition 15 (Kettle Logic-Legal Utterances in Expansion Chain Dialogues). Let $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$ be an expansion chain dialogue. For $0 \leq i \leq n$ we say that “ \mathcal{A}_i ’s utterance at i is kettle-logic legal” iff $i = 0$ or no kettle logic fallacy occurs in $D' = (\{\mathcal{A}_0, \mathcal{A}_i\}, \langle F_0, F_i \rangle, \langle \mathcal{A}_0, \mathcal{A}_i \rangle, t, \sigma)$.

Let us demonstrate how the identification of utterances that are not *kettle-logic legal* (simply referred to as *legal* for the sake of conciseness) can help us avoid kettle logic fallacies. In addition, the below example highlights a phenomenon that we can call *multi-agent kettle logic*.

Example 6 (Example 3 continued). Consider the expansion chain dialogue $D = (\{\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2\}, \langle F_0, F_1, F_2 \rangle, \langle \mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2 \rangle, t, \sigma_{pr})$, where $F_0 = (\{t, a\}, \{\})$, $F_1 = (\{t, a, b\}, \{(a, b), (b, t)\})$, and $F_2 = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$ (see Figure 2; $F_0 = F$, $F_1 = F_b$, and $F_2 = F_{b,c}$). Obviously, the initial utterance of $(\{t, a\}, \{\})$ by \mathcal{A}_0 is legal; we assume that the first move establishes the dialogue’s starting point and hence do not conduct a kettle logic analysis¹¹. \mathcal{A}_1 ’s utterance of

¹¹Such an analysis is possible: in particular, we could construct the expansion chain dialogue $(\{\mathcal{A}_0\}, (\{t\}, \{\}), (\{t, a\}, \{\})), \langle \mathcal{A}_0, \mathcal{A}_0 \rangle, t, \sigma_{pr}$ and check it for kettle logic.

$(\{b\}, \{(a, b), (b, t)\})$ is trivially legal as well because $t \in \sigma_{pref}^\cap(F_1)$. In contrast, \mathcal{A}_2 's utterance of $(\{c\}, \{(b, c), (c, a)\})$ is not legal, which can be easily verified by going back to Example 3. Hence, we can prohibit this utterance, e.g., by stipulating that it is not a “legal move” in our dialogue protocol (see: [22]). Let us highlight here that the kettle logic fallacy is established by several agents: \mathcal{A}_1 and \mathcal{A}_2 . We can accuse neither \mathcal{A}_1 nor \mathcal{A}_2 of uttering inconsistent arguments; instead, it is their joint utterances that lead to a kettle logic fallacy. This is an important observation: in a multi-agent dialogue, we can have agents that use kettle logic to some extent “covertly” in order to jointly defeat a target argument, although these agents may seemingly be in dispute with each other (as the arguments they utter are in conflict).

While this approach is generally applicable, one can argue that is not particularly elegant, because it essentially prohibits inferences drawn by a supposedly “rational” semantics *after the fact*: we react only if we see that an inference is fallacious and tighten the constraints of what is allowed in a topologically arbitrary manner.

5.3 Approach 2: Credulous Kettle-Logic Resistant Extension Selection

If we have a dialogue in which only the credulous kettle logic fallacy occurs, we can mitigate the fallacy by slightly adjusting our inference approach, without violating the constraints imposed by the applied argumentation semantics. Because the approach is straight-forward, we merely demonstrate this using an example¹².

Example 7 (Example 3 continued). Consider the expansion chain dialogue $D = (\{\mathcal{A}_0, \mathcal{A}_1\}, \langle F_0, F_1 \rangle, \langle \mathcal{A}_0, \mathcal{A}_1 \rangle, t, \sigma_{stg})$, where $F_0 = (\{t, a\}, \{\})$ and $F_1 = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$ (see Figure 2; $F_0 = F$ and $F_1 = F_{b,c}$). $\sigma_{stg}(F_0) = \{\{t, a\}\}$ and $\sigma_{stg}(F_1) = \{\{t, a\}, \{b\}, \{t, c\}\}$. Note that we have $\{b\} \in \sigma_{stg}(F_1)$ and $t \notin \{b\}$ and $\forall E \in \sigma_{stg}(F_1) \setminus \{b\}$ it holds that $t \in E$. Analogously to Example 3, we have $\sigma_{stg}(F_1 \downarrow_{\{t,a,b\}}) = \{\{t, a\}\}$ and $\sigma_{stg}(F_1 \downarrow_{\{t,a,c\}}) = \{\{t, c\}\}$ (in these cases, stage semantics behaves the same as the classical Dung semantics). Hence, we see that the credulous kettle logic fallacy occurs in D . However, we can mitigate this fallacy by removing $\{b\}$ from $\sigma_{stg}(F_1)$: if we are only allowed to infer $\{t, a\}$ and $\{t, c\}$ from F_1 , we can claim that we have not violated the constraints of our semantics σ_{stg} and our inference process is no longer subjected to a kettle logic fallacy.

The obvious limitations of the approach is that it merely can mitigate credulous kettle logic, *i.e.*, that it cannot guarantee the general mitigation of kettle logic for any of the surveyed semantics, given the results presented in the previous section.

5.4 Approach 3: Prohibiting Cycles

We may speculate that – at least in finite AFs – kettle logic in abstract argumentation is a problem of cycles, *i.e.*, that only allowing the construction of acyclic argumentation graphs ensures an absence of kettle logic.

¹²The approach reflects, to some extent, the approach to principle-based extension-selection presented in [28].

It is well-known that in case of finite sets of arguments, differences between argumentation semantics (with the exception of naive semantics, which may be considered a mere helper for constructing more advanced semantics) boil down to the handling of cycles. For acyclic finite argumentation frameworks, other semantics yield exactly one extension, which defends all arguments it contains and attacks all arguments that it does not contain.

Corollary 8. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, stg, CF2, stg2\}$. For every $F = (AR, AT) \in \mathcal{FN}$, there exists exactly one σ_x -extension E s.t. $\forall a \in AR$:*

1. $a \in E$, if $\forall b \in AR$ s.t. $(b, a) \in AT$ it holds that E attacks b .
2. $a \notin E$ and $a \in E^+$, otherwise.

The reader may intuitively verify the above observation by following the steps below, given any of the specified argumentation semantics σ_x and an acyclic finite AF $F = (AR, AT)$ (note the algorithm-style notation, where \leftarrow denotes variable assignment).

1. Start with the empty set: $E \leftarrow \emptyset$.
2. Update: $E \leftarrow E \cup \{a \mid a \in AR, a \text{ is unattacked in } F\}$.
3. Update: $F \leftarrow F \downarrow_{AR \setminus E^+}$ (note that the update affects $F = (AR, AT)$).
4. If $AR \neq E$, continue at Step 2.
5. Check that generally, E is the only σ_x -extension of the initial argumentation framework (following the definitions of σ_x , i.e., Definitions 1, 3, 4, and 6, not considering the definition of naive semantics).

Now, we can show that none of the surveyed argumentation semantics is vulnerable to any form of kettle logic with respect to acyclic finite argumentation frameworks¹³

Proposition 11. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, na, stg, CF2, stg2\}$. σ_x is not vulnerable to any form of kettle logic w.r.t. \mathcal{FN} .*

Now, it is easy to show that expansion chain dialogues are free from kettle logic if the argumentation frameworks that occur in them are acyclic.

Corollary 9. *Let $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma_x)$ be an expansion chain dialogue and let $x \in \{pr, st, co, gr, wpr, wco, wgr, na, stg, CF2, stg2\}$. If for $0 \leq i \leq n$ it holds that $F_i \in \mathcal{FN}$ then no kettle logic fallacy occurs in D .*

At this point, let us make a straightforward observation about kettle logic and argumentation frameworks whose argument sets may contain an infinite number of arguments. The proof that the surveyed argumentation semantics are not vulnerable to any

¹³Here, we do not cover naive semantics, for which a more general proposition (with respect to \mathcal{FF}) is provided by Proposition 4.

form of kettle logic considering only acyclic AFs relies on Proposition 8, which does not hold true if we expand the scope and include infinite AFs. The reader may consider Example 5.4 in [10] as a counter-example. We leave studying kettle logic in infinite AFs in greater detail as a potential endeavour for future research.

The results presented in this section show that kettle logic can generally be avoided by constructing only acyclic and finite argumentation graphs and is hence aligned with the intuition that abstract argumentation without cycles is trivial (in that determining the extensions of an acyclic and finite AF is a rather straightforward task). Still, limiting argumentation-based reasoning to acyclic and finite argumentation frameworks severely limits its potential, at least in the sense that many results that have been obtained by the argumentation community and that explicitly or implicitly cover the handling of cycles are irrelevant in this context.

6 Kettle Logic and Other Notions of Argument Acceptance

This paper introduces formal definitions of kettle logic fallacies to Dung-style abstract argumentation, showing that all commonly used semantics are subject to the different variants of the fallacy, even in case of AFs without odd-cycles. The focus is on extension-based semantics and the resulting notions of skeptical and credulous acceptance. However, a substantial line of research on abstract argumentation is concerned with the definition and analysis of additional, more nuanced acceptance statuses, in particular in so-called *labelling-based semantics* [13, 14, 35]. In this section, we provide a brief semi-formal overview of how *labelling-based semantics* are affected by kettle logic and outline potential future research in this direction. Given an AF $F = (AR, AT)$, an argumentation semantics σ , an extension $E \in \sigma(F)$, and an argument $a \in AR$ one can say that:

- a is labelled *IN* (in) w.r.t. E if $a \in E$;
- a is labelled *OUT* (out) w.r.t. E if $a \in E^+$ (assuming that E is conflict-free, an argument cannot be both *IN* and *OUT* with respect to E);
- a is labelled *UND* (undecided) w.r.t. E , otherwise.

Then, given the set of extensions $ES = \sigma(AF)$, we can say that a is:

- *strongly accepted* (*sa*) in F (w.r.t. σ) iff $\forall E \in ES$, a is labelled *IN* w.r.t. E ;
- *weakly accepted* (*wa*) in F (w.r.t. σ) iff $\forall E \in ES$, a is labelled *IN* or *UND* w.r.t. E and $\exists E', E'' \in ES$ s.t. a is labelled *IN* w.r.t. E' and *UND* w.r.t. E'' ;
- *undermined borderline* (*ub*) in F (w.r.t. σ) iff $\exists E, E', E'' \in ES$ s.t. a is labelled *IN* w.r.t. E , *UND* w.r.t. E' , and *OUT* w.r.t. E'' ;
- *determined borderline* (*db*) in F (w.r.t. σ) iff $\forall E \in ES$, a is labelled *UND* w.r.t. E ;

- *weakly rejected* (wr) in F (w.r.t. σ) iff $\forall E \in ES$, a is labelled *UND* or *OUT* w.r.t. E and $\exists E', E'' \in ES$ s.t. a is labelled *UND* w.r.t. E' and *OUT* w.r.t. E'' ;
- *strongly rejected* (sr) in F (w.r.t. σ) iff $\forall E \in ES$, a is labelled *OUT* w.r.t. E .

We call $ST = \{sa, wa, ub, db, wr, sr\}$ *acceptance statuses*. Now, we can say that given two acceptance statuses $s, s' \in ST$ a semantics σ is vulnerable to a kettle logic fallacy from s to s' with respect to a class \mathcal{F} of argumentation frameworks iff there exist $F \in \mathcal{F}$, $F = (AR, AT)$, $t \in AR$, $S \subseteq AR$ such that S is not conflict-free and the following statements hold true:

- t is s in $F \downarrow_{AR \setminus S}$ (w.r.t. σ);
- t is s' in F (w.r.t. σ);
- $\nexists S' \subset S$ s.t. S' is conflict-free and t is s' in $F \downarrow_{(AR \setminus S) \cup S'}$ (w.r.t. σ).

Given two examples provided in Section 4, we can see that stable, (weakly) preferred, (weakly) complete, stage, CF2, and stage2 semantics are vulnerable to a kettle logic fallacy from *strongly accepted* to *strongly rejected* with respect to \mathcal{FF} , \mathcal{FS} , and \mathcal{FO} (consider the example provided in Figure 5) and that grounded and weakly grounded semantics are vulnerable to a kettle logic fallacy from *strongly accepted* to *determined borderline*, again with respect to \mathcal{FF} , \mathcal{FS} , and \mathcal{FO} (Figure 6). However, there are interesting open questions that can be explored, in particular regarding the mitigation of kettle logic. For example, can we avoid kettle logic by employing grounded semantics and stipulating that the strong rejection of an argument is required as a compelling cause for changing our inference in an argumentation dialogue? We leave this and similar questions for future research.

7 Related Research

The analysis that this paper presents is related to several lines of formal argumentation research. Self-evidently, our work is concerned with the principle-based analysis of argumentation semantics as comprehensively covered in [3, 32], which is evidenced by the use of argumentation principles as facilitators of our formal analysis. Also, vulnerability to a form of kettle logic (or absence thereof) can be considered an argumentation principle; here, it is worth highlighting that kettle logic vulnerability is a “negative” principle whose satisfaction is assumed to be undesirable.

The formalisation of kettle logic takes a *dynamic* view on principal-based analysis, and hence is related to the study of argumentation dynamics [18], *i.e.*, to the formal study of change in argumentation frameworks¹⁴. While we do not model change using explicit operators (as for example, done in [15, 11]), we have shown that our approach to modelling dynamics in argumentation dialogues can be characterised as drawing

¹⁴On a related note, our work contributes to the study of argumentation dialogues [12], although we abstain from introducing formal notions for agents or their utterances, for the sake of conciseness.

inferences from an argumentation framework and its *normal expansions* [6], where arguments and attacks are added, but no arguments are removed, and the attacks between previously existing arguments remain unaffected (neither addition, nor removal). In the context of argumentation dynamics, our work focuses on the conditions under which an argument can be forced to be excluded from a conclusion (one or all extensions that a semantics yields). The general line of research that focuses on this problem is the study of *enforcement* [9]. However, the study of how the advancements of conflicting arguments by an agent can affect the acceptance of an argument is apparently novel, with the important exceptions of a study of the role of self-attacking arguments [5] as well as of a study of the effect of the removal of argument on previously inferred extensions [10]. The former work’s main subject can be considered a special case of Kettle logic and is covered by our formal analysis: we distinguish between the class of all argumentation frameworks and the class of argumentation frameworks without self-attacking arguments to highlight that kettle logic is not limited to argumentation frameworks with self-attacking arguments. The latter work considers arguments that (directly or indirectly) attack an extension, but does not focus on the removal of non-conflict-free sets of arguments and hence is different in scope. Finally, in Section 5, we show how kettle logic can be mitigated in some cases through cautious extension selection; this approach is similar to the dynamic enforcement of principles as presented in [28]. However, in the present work, we focus on a different argumentation principle and the introduced mitigation approach is merely a by-product of the presented research, whereas principle enforcement is at the core of [28].

8 Discussion

Conceptually, the results presented in this paper raise the question whether the intuition that the advancement of inconsistent arguments by an agent with one particular dialectical goal is generally problematic and inferior to the advancement of only consistent arguments is correct. Intuitively, one would expect an eloquent agent to advance a consistent set of arguments that carefully and precisely rebuts a given target argument. Our findings indicate, however, that in abstract argumentation, an agent’s only way to reach its dialectical goal is, in some cases, the advancement of arguments that are mutually inconsistent. The results allow for two alternative interpretations with respect to their broader implications on either kettle logic or abstract argumentation.

Questioning Kettle Logic. Proponents of abstract argumentation may claim that kettle logic is not generally a fallacy. After all, it is not as well known as the more classical fallacies that are commonly featured in encyclopediae (*e.g.* [25]) or popular textbooks (*e.g.* [16])¹⁵. Also, one may argue that kettle logic only occurs in cyclic argumentation frameworks, which are, colloquially speaking, “hard” to reason about; hence, for such frameworks logically rational inference may *justifiably* not be fully aligned with all human intuitions about non-fallacious argumentation. Following this line of reasoning, the results presented in this paper

¹⁵We assume that the textbook cited in the introduction that covers the kettle logic fallacy [1] represents an exception and not the rule.

are still useful, because they shed light on the fact that informal human intuitions about argumentation are not aligned with formal argumentation approaches and that the former can be advanced based on results obtained from studying the latter. Finally, one can argue that analysing reasoning fallacies such as kettle logic in formal argumentation should consider the structure of arguments in order to avoid getting lost in abstraction [31]. Indeed, an analysis of kettle logic in the context of structured argumentation approaches such as ASPIC+ [30] and Assumption-Based Argumentation (ABA) [20] may be interesting future work. However, let us argue that in the case of our analysis of the kettle logic fallacy, the results are particularly appealing because they show that even abstract argumentation as the most fundamental (and also very simple) approach to computational models of arguments is vulnerable to kettle logic; also, from this result it follows that approaches extending abstract argumentation (such as ASPIC+ and ABA) are subject to the fallacy as well.

Questioning the Rationality of Abstract Argumentation. Taking a more critical stance on abstract argumentation research, one can argue that the often purely technical focus of argumentation semantics design and analysis, guided by the intuitions of the logicians who conduct it, places a too strong focus on classical semantics, although the behaviour of these semantics is aligned neither with behaviour that is formally expected from a broader, cross-disciplinary perspective, nor with human intuition. Indeed, Dung’s classical semantics all violate the *weak reference independence principle* that is equivalent to the *consistent preferences* property of economic rationality [27] (the arguably most well-known formal principle of “rational” reasoning and decision-making). Weak reference independence is, however, satisfied by CF2 semantics, which also fairs better than classical, admissible set-based semantics in human (non-expert) evaluations of argumentation semantics [24]. This, one can argue, points to a bias in argumentation research: although the community spends substantial efforts on abstract argumentation research, which in turn amounts to research on the handling of (odd) cycles, the way that cycles are handled in most research endeavours is misaligned with insights that broader perspectives – based on insights from philosophy, psychology, or economics – can provide. Still, in the context of kettle logic, it seems obvious, from the conducted principle-based analysis, as well as from the provided counter-examples that a “reasonable” semantics that is not vulnerable to the different forms of kettle logic is hard or even impossible to design.

We leave it to the reader to decide whether to interpret the findings conservatively, as in the former line of argumentation, or more provocatively, as in the latter.

9 Conclusion

In this paper, we have formalised the intuition of kettle logic for formal argumentation, and shown that the advancement of inconsistent arguments can be beneficial for an agent that wants to reject a particular target argument. The findings highlight that

the phenomenon of successful persuasion by the advancement of inconsistent arguments does not only apply to human argumentation in highly complex social settings, but also to the supposedly “rational” inference functions that are applied to infer sets of arguments from simple directed argumentation graphs. Future research can potentially analyse kettle logic in the context of more sophisticated formal argumentation approaches, as well as to empirically study human kettle logic argumentation fallacies in the wild in order to strengthen the bridge between human and formal argumentation.

Acknowledgements

The author thanks the anonymous reviewers for their thoughtful and detailed comments that have greatly improved the presentation of the paper.

Appendix

The appendix re-states all corollaries, lemmata, and propositions and provides their proofs.

Proposition 1. *If an argumentation semantics σ is vulnerable to rejection kettle logic, it is also vulnerable to credulous rejection kettle logic.*

Proof. For every $F = (AR, AT)$ in a class of AFs \mathcal{F} , for every argumentation semantics σ , it holds true for $t \in AR$ and $S \subseteq AR$ that if $t \in \sigma^\cap(F \downarrow_{AR \setminus S})$ then $t \in \sigma^\cup(F \downarrow_{AR \setminus S})$. Considering this implication relationship, the proof follows directly from the definitions of rejection kettle logic and credulous rejection kettle logic (Definition 11). \square

Proposition 2. *Every argumentation semantics σ that satisfies admissibility and weak reinstatement is vulnerable to all forms of kettle logic.*

Proof. Consider the AFs $F = (\{t, a\}, \{\})$ and $F_{b,c} = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$ (Figures 2a and 2d), and the target argument t . Because σ satisfies weak reinstatement, it must hold that $\sigma(F) = \{\{t, a\}\}$. Because σ satisfies admissibility, it must hold that $\sigma(F_{b,c}) = \{\{\}\}$ or $\sigma(F_{b,c}) = \{\}$. So $\forall E \in \sigma(F)$ it holds that $t \in E$ but $\nexists E' \in \sigma(F_{b,c})$ s.t. $t \in E'$. In contrast, because σ satisfies weak reinstatement, it holds, for $F_b = F_{b,c} \downarrow_{\{t, a, b\}}$ and $F_c = F_{b,c} \downarrow_{\{t, a, c\}}$, that $\forall E'' \in \sigma(F_b) \cup \sigma(F_c)$, $t \in E''$ (see Figures 2b and 2c). Hence, we have the following cases, considering Definition 11, as well as $S = \{b, c\}$ (note that S is not conflict-free) and $F' = (AR', AT') = F_{b,c}$.

1) The proof that σ is vulnerable to rejection kettle logic. The following statements hold true: i) $t \in \sigma^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma^\cup(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cup(F' \downarrow_{(AR \setminus S) \cup S'})$. This proves the proposition for this case.

2) The proof that σ is vulnerable to credulous kettle logic. The following statements hold true: i) $t \in \sigma^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma^\cap(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cap(F' \downarrow_{(AR \setminus S) \cup S'})$. This proves the proposition for this case.

3) The proof that σ is vulnerable to credulous rejection kettle logic follows from 1) and Proposition 1. \square

Corollary 1. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr\}$. σ_x is vulnerable to all forms of kettle logic.*

Proof. The proof follows directly from the example depicted in Figure 1 (note that $\sigma_{st}(F_{b,c}) = \{\}$). Alternatively, we can point to [2]: σ_x satisfies weak reinstatement. Because σ_x also satisfies admissibility, the proof follows directly from Proposition 2. \square

Corollary 2. *Every argumentation semantics σ that satisfies admissibility and weak reinstatement is vulnerable to all forms of kettle logic w.r.t. FS .*

Proof. Again (analogously to the proof of Proposition 2), the proof follows directly from the example depicted by Figure 2, because for the AF F depicted in the figure, it holds true that $F \in \mathcal{FS}$. \square

Corollary 3. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr\}$. σ_x is vulnerable to all forms of kettle logic w.r.t. \mathcal{FS} .*

Proof. Again, we can point to [2]: σ_x satisfies weak reinstatement. Because σ_x also satisfies admissibility, the proof follows directly from Proposition 2. \square

Proposition 3. *Every argumentation semantics σ that satisfies weak admissibility and weak reinstatement is vulnerable to all forms of kettle logic.*

Proof. Consider the AFs $F^* = (\{t, a, b\}, \{(a, b)\})$ and $F_{c,d}^* = (\{t, a, b, c, d\}, \{(a, b), (a, c), (b, d), (c, t), (d, a)\})$ (Figures 4a and 4d), and the target argument t . Because σ satisfies weak reinstatement and weak admissibility, it must hold that $\sigma(F^*) = \{\{t, a\}\}$ and that $t \notin \sigma^\cup(F_{c,d}^*)$. So, $\forall E \in \sigma(F^*)$ it holds that $t \in E$ but $\nexists E' \in \sigma(F_{c,d}^*)$ s.t. $t \in E'$. In contrast, because σ satisfies weak reinstatement, it must hold, for $F_c^* = F_{c,d}^* \downarrow_{\{t,a,b,c\}}$ and $F_d^* = F_{c,d}^* \downarrow_{\{t,a,b,d\}}$, that $\forall E'' \in \sigma(F_c^*) \cup \sigma(F_d^*)$, $t \in E''$ (see Figures 4b and 4c). Hence, we have the following cases, considering Definition 11, as well as $S = \{c, d\}$ (note that S is not conflict-free) and $F' = (AR', AT') = F_{c,d}^*$.

1) The proof that σ is vulnerable to rejection kettle logic. The following statements hold true: i) $t \in \sigma^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma^\cup(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma^\cup(F' \downarrow_{(AR' \setminus S) \cup S'})$. This proves the proposition for this case.

2) The proof that σ is vulnerable to credulous kettle logic. The following statements hold true: i) $t \in \sigma^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma^\cap(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $(t \notin \sigma^\cap(F' \downarrow_{(AR' \setminus S) \cup S'}))$. This proves the proposition for this case.

3) The proof that σ is vulnerable to credulous rejection kettle logic follows from 1) and Proposition 1. \square

Corollary 4. *Let σ_x be an argumentation semantics s.t. $x \in \{wpr, wco, wgr\}$. σ_x is vulnerable to all forms of kettle logic.*

Proof. The proof follows directly from the example depicted in Figure 4. Alternatively, we can point to [8]: σ_x satisfies weak reinstatement. Because σ_x also satisfies weak admissibility, the proof follows directly from Proposition 3. \square

Corollary 5. *Let σ_x be an argumentation semantics s.t. $x \in \{wpr, wco, wgr\}$. σ_x is vulnerable to all forms of kettle logic w.r.t. \mathcal{FS} .*

Proof. Again, we can point to [8]: σ_x satisfies weak reinstatement. Because σ_x also satisfies weak admissibility, the proof follows directly from Proposition 3 and the example depicted in Figure 4. \square

Proposition 4. *σ_{na} is not vulnerable to any form of kettle logic.*

Proof. Consider any AFs $F = (AR, AT)$ and $F' = (AR', AT')$ s.t. it holds true that an argument t is in AR and $F' \downarrow_{AR \setminus S} = F$ for some $S \subseteq AR' \setminus AR$. By definition of σ_{na} , the following statements hold true:

1. Iff t is self-attacking then $t \notin \sigma_{na}^{\cup}(F)$;
2. Iff t is self-attacking then $t \notin \sigma_{na}^{\cup}(F')$;
3. If t is not self-attacking, then $t \in \sigma_{na}^{\cup}(F)$ and $t \in \sigma_{na}^{\cup}(F')$ (follows from Statements 1. and 2.);
4. Iff $t \notin \sigma_{na}^{\cup}(F)$ then $t \notin \sigma_{na}^{\cup}(F')$ (follows from Statements 1. and 2.);
5. $t \in \sigma_{na}^{\cap}(F)$ and $t \notin \sigma_{na}^{\cap}(F')$ iff no argument $AR \setminus \{a \mid a \text{ is self-attacking}\}$ attacks t or is attacked by t (in F) and at least one argument $AR' \setminus \{a \mid a \text{ is self-attacking}\}$ attacks t or is attacked by t .

From Statements 1., 2., and 3. it follows that σ_{na} is not vulnerable to credulous rejection kettle logic. From Statement 4. it follows that σ_{na} is not vulnerable to rejection kettle logic. From Statement 5. it follows that if $t \in \sigma_{na}^{\cap}(F)$ and $t \notin \sigma_{na}^{\cap}(F')$ then, by definition of σ_{na} , it holds that $\exists a \in AR' \setminus AR$ s.t. a is not self-attacking and $t \notin \sigma_{na}^{\cap}(F' \downarrow_{AR \cup \{a\}})$ and consequently, σ_{na} is not vulnerable to credulous kettle logic. \square

Proposition 5. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic.*

Proof. Consider the AFs $F = (\{t, a\}, \{\})$ and $F_{b,c} = (\{t, a, b, c\}, \{(a, b), (b, c), (b, t), (c, a)\})$ (Figures 2a and 2d), and the target argument t . We have $\sigma_x(F) = \{\{t, a\}\}$, $\sigma_x(F_{b,c}) = \{\{t, a\}, \{b\}, \{t, c\}\}$. So $\forall E \in \sigma_x(F)$ it holds that $t \in E$ but $\exists E' \in \sigma_x(F_{b,c})$ s.t. $t \notin E'$. In contrast, it holds, for $F_b = F_{b,c} \downarrow_{\{t, a, b\}}$ and $F_c = F_{b,c} \downarrow_{\{t, a, c\}}$, that $\sigma_x(F_b) = \{\{t, a\}\}$ and $\sigma_x(F_c) = \{\{t, c\}\}$ and hence that $\forall E'' \in \sigma_x(F_b) \cup \sigma_x(F_c)$, $t \in E''$ (see Figures 2b and 2c). Consequently, it follows from the definition of kettle logic (Definition 11) that σ_x is vulnerable to credulous kettle logic. \square

Corollary 6. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic w.r.t. \mathcal{FS} .*

Proof. The proof is analogous to the proof of Proposition 5. We merely need to observe that $F, F_b, F_c, F_{b,c} \in \mathcal{FS}$ (see Figure 2). \square

Proposition 6. *Let σ_x be an argumentation semantics s.t. $x \in \{stg, CF2, stg2\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic.*

Proof. Consider the AFs $F^* = (\{t, a, b\}, \{(a, b), (b, a)\})$ and $F_{c,d}^* = (\{t, a, b, c, d\}, \{(a, b), (a, c), (b, a), (b, d), (c, a), (c, d), (c, t), (d, b), (d, c), (d, t)\})$ (Figures 5a and 4d), and the target argument t . It holds that $\sigma_x(F^*) = \{\{a, t\}, \{b, t\}\}$ and that $t \notin \sigma_x^{\cup}(F_{c,d}^*)$. So, $\forall E \in \sigma_x(F^*)$ it holds that $t \in E$ but $\nexists E' \in \sigma_x(F_{c,d}^*)$ s.t. $t \in E'$. In contrast, it holds, for $F_c^* = F_{c,d}^* \downarrow_{\{t, a, b, c\}}$ and $F_d^* = F_{c,d}^* \downarrow_{\{t, a, b, d\}}$, that $\exists E'' \in \sigma_x(F_c^*)$ s.t. $t \in E''$ and $\exists E^{**} \in \sigma_x(F_d^*)$ s.t. $t \in E^{**}$ (see Figures 5b and 5c). Hence, we have the following cases, considering Definition 11, as well as $S = \{c, d\}$ (note that S is not conflict-free) and $F' = (AR', AT') = F_{c,d}^*$.

- 1) The proof that σ_x is vulnerable to rejection kettle logic.** The following statements hold true: i) $t \in \sigma_x^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma_x^\cup(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma_x^\cup(F' \downarrow_{(AR \setminus S) \cup S'})$. This proves the proposition for this case.
- 2) The proof that σ_x is vulnerable to credulous rejection kettle logic.** The following statements hold true: i) $t \in \sigma_x^\cup(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma_x^\cup(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma_x^\cup(F' \downarrow_{(AR \setminus S) \cup S'})$. This proves the proposition for this case.

□

Proposition 7. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, wpr, wco, stg, CF2, stg2\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic w.r.t. \mathcal{FO} .*

Proof. The proof is analogous to the proof of Proposition 6. (Note that there are difference in semantics behaviour for the counter-example, but still, the same counter-example applies to all semantics.) We merely need to observe that $F^*, F_c^*, F_d^*, F_{c,d}^* \in \mathcal{FO}$ (see Figure 5). □

Proposition 8. *Let σ_x be an argumentation semantics s.t. $x \in \{wgr, gr\}$. σ_x is vulnerable to rejection kettle logic and credulous rejection kettle logic w.r.t. \mathcal{FO} .*

Proof. Consider the AFs $F^* = (\{t, a, b, c\}, \{(a, t), (b, a), (c, a)\})$ and $F_{d,e}^* = (\{t, a, b, c, d, e\}, \{(a, t), (b, a), (c, a), (d, b), (d, e), (e, c), (e, d)\})$ (Figures 6a and 6d), and the target argument t . We observe that $\sigma_x(F^*) = \{\{t, b, c\}\}$ and $t \notin \sigma_x^\cup(F_{d,e}^*)$. So, $\forall E \in \sigma_x(F^*)$ it holds that $t \in E$ but $\nexists E' \in \sigma_x(F_{d,e}^*)$ s.t. $t \in E'$. In contrast, for $F_d^* = F_{d,e}^* \downarrow_{\{t, a, b, c, d\}}$ and $F_e^* = F_{d,e}^* \downarrow_{\{t, a, b, c, e\}}$ it holds that $\forall E'' \in \sigma_x(F_d^*) \cup \sigma_x(F_e^*)$, $t \in E''$ (see Figures 6b and 6c). We observe that $F^*, F_{d,e}^*, F_d^*, F_e^* \in \mathcal{FO}$. Hence, we have the following cases, considering Definition 11, as well as $S = \{d, e\}$ (note that S is not conflict-free) and $F' = (AR', AT') = F_{d,e}^*$.

- 1) The proof that σ_x is vulnerable to rejection kettle logic w.r.t. \mathcal{FO} .** The following statements hold true: i) $t \in \sigma_x^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma_x^\cup(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma_x^\cup(F' \downarrow_{(AR \setminus S) \cup S'})$. This proves the proposition for this case.
- 2) The proof that σ_x is vulnerable to credulous rejection kettle logic w.r.t. \mathcal{FO}** follows from 1) and Proposition 1.

□

Proposition 9. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, stg, CF2, stg2\}$. σ_x is vulnerable to credulous kettle logic w.r.t. \mathcal{FO} .*

Proof. Consider the AFs $F^* = (\{t, a, b, c\}, \{(a, t), (b, a), (c, a)\})$ and $F_{d,e}^* = (\{t, a, b, c, d, e\}, \{(a, t), (b, a), (c, a), (d, b), (d, e), (e, c), (e, d)\})$ (Figures 6a and 6d), and the target argument t . We observe that $\sigma_x(F^*) = \{\{t, b, c\}\}$ and $t \notin \sigma_x^\cap(F_{d,e}^*)$. In contrast, for $F_d^* = F_{d,e}^* \downarrow_{\{t, a, b, c, d\}}$ and $F_e^* = F_{d,e}^* \downarrow_{\{t, a, b, c, e\}}$ it holds that $\forall E'' \in \sigma_x(F_d^*) \cup \sigma_x(F_e^*)$, $t \in E''$ (see Figures 6b and 6c). We observe that $F^*, F_{d,e}^*, F_d^*, F_e^* \in \mathcal{FO}$.

Hence, we can observe the following, considering Definition 11, as well as $S = \{d, e\}$ (note that S is not conflict-free) and $F' = (AR', AT') = F_{d,e}^*$: i) $t \in \sigma_x^\cap(F' \downarrow_{AR' \setminus S})$; ii) $t \notin \sigma_x^\cap(F')$; iii) $\nexists S' \subset S$ s.t. S' is conflict-free and $(t \notin \sigma_x^\cap(F' \downarrow_{(AR \setminus S) \cup S'}))$. This proves that σ_x is vulnerable to credulous kettle logic w.r.t \mathcal{FO} . \square

Lemma 1. *An argumentation semantics σ is:*

1. *vulnerable to chain rejection kettle logic iff it is vulnerable to rejection kettle logic;*
2. *vulnerable to chain credulous kettle logic iff it is vulnerable to credulous kettle logic;*
3. *vulnerable to chain credulous rejection kettle logic iff it is vulnerable to credulous rejection kettle logic.*

Proof. Consider the definitions of kettle logic (Definition 11) and chain kettle logic (Definition 12). We merely need observe that (considering the definition of a normal expansion, i.e., Definition 10), given an AF $F = (AR, AT)$ and a set of arguments $S \subseteq AR$ it holds that $F \downarrow_{AR \setminus S} \preceq_N F$ and given $S' \subset S$ it holds that $F \downarrow_{AR \setminus S} \preceq_N F \downarrow_{(AR \setminus S) \cup S'}$ and $F \downarrow_{(AR \setminus S) \cup S'} \preceq_N F$. \square

Corollary 7. *If a rejection/credulous/credulous rejection kettle logic fallacy occurs in an expansion chain dialogue $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$ then σ is vulnerable to rejection/credulous/credulous rejection kettle logic.*

Proof. The proof follows directly from the definitions of kettle logic, chain kettle logic, and kettle logic in expansion chain dialogues (Definitions 11, 12 and 14) and Lemma 1. \square

Proposition 10. *If a rejection kettle logic fallacy occurs in an expansion chain dialogue D , then a credulous rejection kettle logic fallacy occurs in D .*

Proof. Consider the definition of kettle logic in expansion chain dialogues (Definition 14). For every expansion chain dialogue $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma)$, a rejection kettle logic fallacy occurs in D iff for some AFs $F_0, F_i, 0 < i \leq n$, $F_0 = (AR_0, AT_0)$, $F_i = (AR_i, AT_i)$ s.t. $AR_i \setminus AR_0$ is not conflict-free, the following statements hold true:

1. $t \in \sigma^\cap(F_0)$;
2. $t \notin \sigma^\cup(F_i)$;
3. $\nexists F' \in \mathcal{F}, F' = (AR', AT')$ s.t. $F_0 \preceq_N F', F' \preceq F_i, AR' \setminus AR_0$ is conflict-free, and $t \notin \sigma^\cup(F')$.

Because $t \in \sigma^\cap(F_0)$ (Statement 1) implies that $t \in \sigma^\cup(F_0)$, it follows trivially from the definition of credulous rejection kettle logic in expansion chains that if a rejection kettle logic fallacy occurs in D then a credulous rejection kettle logic fallacy occurs in D . This proves the proposition. \square

Corollary 8. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, stg, CF2, stg2\}$. For every $F = (AR, AT) \in \mathcal{FN}$, there exists exactly one σ_x -extension E s.t. $\forall a \in AR$:*

1. $a \in E$, if $\forall b \in AR$ s.t. $(b, a) \in AT$ it holds that E attacks b .
2. $a \notin E$ and $a \in E^+$, otherwise.

Proof. For every $F \in \mathcal{FN}$ we observe that:

1. If there exists exactly one stable extension for F , the to-be-proven statement is trivially true for stable semantics.
2. $|\sigma_{wpr}(F)| = |\sigma_{wco}(F)| = |\sigma_{wgr}(F)| = 1$, which is shown in [7] (Proposition 6.5);
3. $\sigma_{wpr}(F) = \sigma_{pr}(F)$, $\sigma_{wco}(F) = \sigma_{co}(F)$, and $\sigma_{wgr}(F) = \sigma_{gr}(F)$, which is shown in [7] (Corollary 6.6);
4. $\sigma_{st}(F) = \sigma_{pr}(F) = \sigma_{co}(F) = \sigma_{gr}(F) = \sigma_{CF2}(F)$, which is shown in [2] (Proposition 5.2);
5. $\sigma_{st}(F) = \sigma_{stg}(F)$ and $\sigma_{CF2}(F) = \sigma_{stg2}(F)$ can be trivially verified from their definitions (Definitions 1, 4, and 6).

The proof follows directly from the above statements, as we know that there is exactly one stable extension for F and thus the proof holds for stable semantics and the extensions of other semantics are equivalent to stable extensions given $F \in \mathcal{FN}$. \square

Proposition 11. *Let σ_x be an argumentation semantics s.t. $x \in \{st, pr, co, gr, wpr, wco, wgr, na, stg, CF2, stg2\}$. σ_x is not vulnerable to any form of kettle logic w.r.t. \mathcal{FN} .*

Proof. From Corollary 8, we know that for every $F \in \mathcal{FN}$ it holds that $|\sigma_x(F)| = 1$. Hence, by definition of kettle logic (Definition 11), it follows that σ_x is vulnerable to credulous kettle logic and credulous rejection kettle logic w.r.t. \mathcal{FN} iff σ_x is vulnerable to rejection kettle logic w.r.t. \mathcal{FN} . Given an AF $F^* \in \mathcal{FN}$, let us denote the only σ_x -extension of F^* by E_{F^*} . By definition of rejection kettle logic, σ_x is vulnerable to rejection kettle logic w.r.t. \mathcal{FN} iff $\exists F \in \mathcal{FN}$, $F = (AR, AT)$, and $\exists t \in AR$, $S \subseteq AR$ s.t. S is conflict-free and the following statements hold true:

- $t \in \sigma_x^\cap(F \downarrow_{AR \setminus S})$;
- $t \notin \sigma_x^\cup(F)$;
- $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma_x^\cup(F \downarrow_{(AR \setminus S) \cup S'})$.

Assume for a contradiction that this is the case. From Corollary 8 we know that because for every $F' \in \mathcal{FN}$, $F' = (AR', AT')$, $E_{F'}$ contains all arguments it defends (and only these) and attacks all arguments in $AR' \setminus E_{F'}$. Consequently, it must hold that $t \notin E_{F'}$, where $F'' = F \downarrow_{(AR \setminus S) \setminus S''}$ and $S'' = \{a \mid a \in S, E_{F'} \text{ attacks } a\}$. It follows that $\nexists S' \subset S$ s.t. S' is conflict-free and $t \notin \sigma_x^\cup(F \downarrow_{(AR \setminus S) \cup S'})$ does not hold true, which establishes the contradiction and proves the proposition. \square

Corollary 9. *Let $D = (\mathcal{AS}, \langle F_0, \dots, F_n \rangle, \langle \mathcal{A}_0, \dots, \mathcal{A}_n \rangle, t, \sigma_x)$ be an expansion chain dialogue and let $x \in \{pr, st, co, gr, wpr, wco, wgr, na, stg, CF2, stg2\}$. If for $0 \leq i \leq n$ it holds that $F_i \in \mathcal{FN}$ then no kettle logic fallacy occurs in D .*

Proof. The proof follows from Corollary 8 (σ_x s.t. $x \in \{pr, st, co, gr, wpr, wco, wgr, stg, CF2, stg2\}$ is not vulnerable to any form of kettle logic w.r.t. \mathcal{FN}), Proposition 4 (σ_{na} is not vulnerable to any form of kettle logic), and Corollary 7 (if a kettle logic fallacy occurs in D then σ_x must be vulnerable to a form of kettle logic). \square

References

- [1] Arp, R., Barbone, S., Bruce, M.: Bad arguments: 100 of the most important fallacies in Western Philosophy. John Wiley & Sons (2018)
- [2] Baroni, P., Caminada, M., Giacomin, M.: Abstract argumentation frameworks and their semantics. In: Baroni, P., Gabbay, D., Massimiliano, G., van der Torre, L. (eds.) Handbook of Formal Argumentation. College Publications, chap. 4, pp. 159–236. College Publications (2018)
- [3] Baroni, P., Giacomin, M.: On principle-based evaluation of extension-based argumentation semantics. Artificial Intelligence **171**(10-15), 675–700 (2007). <https://doi.org/10.1016/j.artint.2007.04.004>
- [4] Baroni, P., Giacomin, M., Guida, G.: SCC-recursiveness: a general schema for argumentation semantics. Artif. Intell. **168**(1-2), 162–210 (2005). <https://doi.org/10.1016/j.artint.2005.05.006>
- [5] Baumann, R., Woltran, S.: The role of self-attacking arguments in characterizations of equivalence notions. Journal of Logic and Computation **26**(4), 1293–1313 (Aug 2016). <https://doi.org/10.1093/logcom/exu010>
- [6] Baumann, R., Brewka, G.: Expanding argumentation frameworks: Enforcing and monotonicity results. COMMA **10**, 75–86 (2010)
- [7] Baumann, R., Brewka, G., Ulbricht, M.: Comparing weak admissibility semantics to their dung-style counterparts - reduct, modularization, and strong equivalence in abstract argumentation. In: Calvanese, D., Erdem, E., Thielscher, M. (eds.) Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, KR 2020, Rhodes, Greece, September 12–18, 2020. pp. 79–88 (2020). <https://doi.org/10.24963/kr.2020/9>
- [8] Baumann, R., Brewka, G., Ulbricht, M.: Shedding new light on the foundations of abstract argumentation: Modularization and weak admissibility. Artificial Intelligence **310**, 103742 (2022). <https://doi.org/https://doi.org/10.1016/j.artint.2022.103742>
- [9] Baumann, R., Doutre, S., Mailly, J.G., Wallner, J.P.: Enforcement in formal argumentation. IfColog Journal of Logics and their Applications (FLAP) **8**(6), 1623–1678 (2021)

- [10] Baumann, R., Ulbricht, M.: On cycles, attackers and supporters - A contribution to the investigation of dynamics in abstract argumentation. In: Zhou, Z. (ed.) Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021, Virtual Event / Montreal, Canada, 19-27 August 2021. pp. 1780–1786. *ijcai.org* (2021). <https://doi.org/10.24963/ijcai.2021/245>, <https://doi.org/10.24963/ijcai.2021/245>
- [11] Bisquert, P., Cayrol, C., Dupin de Saint Cyr Bannay, F., Lagasquie-Schiex, M.C.: Change in argumentation systems: exploring the interest of removing an argument. In: SUM’11. pp. 275–288 (2011)
- [12] Black, E., Maudet, N., Parsons, S.: Argumentation-based dialogue. In: Handbook of Formal Argumentation, Volume 2, pp. 511–575. College Publications (2021)
- [13] Caminada, M.: On the issue of reinstatement in argumentation. In: Fisher, M., van der Hoek, W., Konev, B., Lisitsa, A. (eds.) Logics in Artificial Intelligence, 10th European Conference, JELIA 2006, Liverpool, UK, September 13-15, 2006, Proceedings. Lecture Notes in Computer Science, vol. 4160, pp. 111–123. Springer (2006). https://doi.org/10.1007/11853886_11, https://doi.org/10.1007/11853886_11
- [14] Caminada, M.W.A., Gabbay, D.M.: A logical account of formal argumentation. *Stud Logica* **93**(2-3), 109–145 (2009). <https://doi.org/10.1007/s11225-009-9218-x>, <https://doi.org/10.1007/s11225-009-9218-x>
- [15] Cayrol, C., de Saint-Cyr, F.D., Lagasquie-Schiex, M.C.: Change in abstract argumentation frameworks: Adding an argument. *Journal of Artificial Intelligence Research* **38**, 49–84 (2010)
- [16] Copi, I., Cohen, C., Rodych, V.: Introduction to Logic. Taylor & Francis (2018)
- [17] Derrida, J.: Resistances of Psychoanalysis. Stanford University Press (2022). <https://doi.org/doi:10.1515/9781503616707>
- [18] Doutre, S., Maily, J.G.: Constraints and changes: A survey of abstract argumentation dynamics. *Argument & Computation* **9**, 223–248 (2018). <https://doi.org/10.3233/AAC-180425>
- [19] Dung, P.M.: On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artificial intelligence* **77**(2), 321–357 (1995)
- [20] Dung, P.M., Kowalski, R.A., Toni, F.: Assumption-based argumentation. In: Simari, G.R., Rahwan, I. (eds.) *Argumentation in Artificial Intelligence*, pp. 199–218. Springer (2009). https://doi.org/10.1007/978-0-387-98197-0_10, https://doi.org/10.1007/978-0-387-98197-0_10
- [21] Dvořák, W., Gaggl, S.A.: Stage semantics and the SCC-recursive schema for argumentation semantics. *Journal of Logic and Computation* **26**(4), 1149–1202 (02 2014). <https://doi.org/10.1093/logcom/exu006>

- [22] Fan, X., Toni, F.: Agent strategies for aba-based information-seeking and inquiry dialogues. In: Raedt, L.D., Bessiere, C., Dubois, D., Doherty, P., Frasconi, P., Heintz, F., Lucas, P.J.F. (eds.) ECAI 2012 - 20th European Conference on Artificial Intelligence. Including Prestigious Applications of Artificial Intelligence (PAIS-2012) System Demonstrations Track, Montpellier, France, August 27-31, 2012. *Frontiers in Artificial Intelligence and Applications*, vol. 242, pp. 324–329. IOS Press (2012). <https://doi.org/10.3233/978-1-61499-098-7-324>
- [23] Freud, S.: *Die Traumdeutung*. Fischer Taschenbuch Verlag (2003), Frankfurt am Main (1899)
- [24] Guillaume, M., Cramer, M., van der Torre, L., Schiltz, C.: Reasoning on conflicting information: An empirical study of formal argumentation. *PLOS ONE* **17**(8), 1–21 (08 2022). <https://doi.org/10.1371/journal.pone.0273225>
- [25] Hansen, H.: Fallacies. In: Zalta, E.N. (ed.) *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2020 edn. (2020)
- [26] Kakas, A., Mancarella, P.: On the semantics of abstract argumentation. *Journal of Logic and Computation* **23**(5), 991–1015 (01 2013). <https://doi.org/10.1093/logcom/exs068>
- [27] Kampik, T., Nieves, J.C.: Abstract argumentation and the rational man. *Journal of Logic and Computation* **31**(2), 654–699 (02 2021). <https://doi.org/10.1093/logcom/exab003>
- [28] Kampik, T., Nieves, J.C., Gabbay, D.: Ensuring reference independence and cautious monotony in abstract argumentation. *International Journal of Approximate Reasoning* **140**, 173–210 (2022). <https://doi.org/https://doi.org/10.1016/j.ijar.2021.10.007>
- [29] Mayer, S., Ajanovic, E., Sauer, B.: Intersections and inconsistencies. framing gender in right-wing populist discourses in Austria. *NORA - Nordic Journal of Feminist and Gender Research* **22**(4), 250–266 (2014). <https://doi.org/10.1080/08038740.2014.964309>
- [30] Modgil, S., Prakken, H.: The *ASPIC*⁺ framework for structured argumentation: a tutorial. *Argument Comput.* **5**(1), 31–62 (2014). <https://doi.org/10.1080/19462166.2013.869766>, <https://doi.org/10.1080/19462166.2013.869766>
- [31] Prakken, H., Winter, M.D.: Abstraction in argumentation: Necessary but dangerous. In: Modgil, S., Budzynska, K., Lawrence, J. (eds.) *Computational Models of Argument - Proceedings of COMMA 2018*, Warsaw, Poland, 12-14 September 2018. *Frontiers in Artificial Intelligence and Applications*, vol. 305, pp. 85–96. IOS Press (2018). <https://doi.org/10.3233/978-1-61499-906-5-85>, <https://doi.org/10.3233/978-1-61499-906-5-85>

- [32] van der Torre, L., Vesic, S.: The principle-based approach to abstract argumentation semantics. *IfCoLog Journal of Logics and Their Applications* **4**(8) (October 2017)
- [33] Verheij, B.: Two approaches to dialectical argumentation: admissible sets and argumentation stages. *Proc. NAIC* **96**, 357–368 (1996)
- [34] Wible, A.: *Kettle Logic*, chap. 34, pp. 174–176. John Wiley & Sons, Ltd (2018). <https://doi.org/https://doi.org/10.1002/9781119165811.ch34>
- [35] Wu, Y., Caminada, M.: A labelling-based justification status of arguments. *Studies in Logic* **3**(4), 12–29 (2010)