Trends and Calendar Effects in Stock Returns

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Abstract

This paper presents statistical investigations regarding the value of the trend concept and calendar effects for prediction of stock returns. The examined data covers 207 stocks on the Swedish stock market for the time period 1987-1996. The results show a very weak trend behaviour. The massive better part of returns falls into a region, where it is very difficult to claim any correlation between past and future price trends.
It is also shown that seasonal variables, such as the month of the year, affect the stock returns more than the average daily returns. This is consequential for all methods, where the seasonal variables are not taken into account in predicting daily stock returns.

1. Introduction

This paper presents results from a statistical analysis of stocks of 207 major stocks from the Swedish stock market for the period 1987-1996. The purpose of the analysis is to examine the concepts of trend and calendar effects, since they are often claimed to exist and are often used in technical analysis.

2. Definitions

In the presentation of statistics we will use a few terms that will be defined in this section. The *k*-step return $R_k(t)$ is defined as the relative increase in price for the previous *k* days:

$$R_k(t) = 100 \cdot \frac{y(t) - y(t-k)}{y(t-k)}. \quad (1)$$

The basic statistical properties of $R_k(t)$ for 207 stocks from the Swedish stock market for the period 1987-1996 are listed in Table 1.

The values in the table are mean values for all stocks. Each column shows data for one particular value of *k*. The last six lines in the tables show the distribution of signs for the returns. "*Return* = 0" is the fraction of returns equal to zero. "*Return* > 0" is the fraction of returns greater than zero and "*Return* < 0" is the fraction of returns less than zero. "*Up fraction*" is defined as:

$$100 \cdot \frac{\text{“}Return > 0\text{”}}{\text{“}Return > 0\text{”} + \text{“}Return < 0\text{”}}, \quad (2)$$

which is the positive fraction of all non-zero moves. "*Up fraction*" is a relevant measure, when it comes to evaluating the hit rate of prediction algorithms. Looking at one-step returns in the tables, the "*Up fraction*" for the for the 207 stocks is 50.6%. The "*Mean Up*" and "*Mean Down*" columns show the mean value of the positive and negative returns respectively. The fractions of zero returns in the data material are somewhat surprisingly high 23.4%. The zero returns must be dealt with in a proper way when evaluating hit rates for prediction algorithms. The "*Up fraction*" circumvents the zero returns by simply removing them before calculating the hit rate. In this way, the zero returns are counted as both increases and decreases, in equal proportions.
We suggest the following definition for the *k*-trend $T_k(t)$:

$$T_k(t) = \frac{100}{k} \cdot \frac{y(t) - y(t-k)}{y(t-k)}. \quad (3)$$

It is convenient to divide by *k* in order to get the daily increase in price. Trend values for different values of *k* can then be analyzed on an equal basis. To see if $T_k(t)$ is connected to future changes, define the profit $P_h(t)$ computed *h* days ahead as:

$$P_h(t) = 100 \cdot \frac{y(t+h) - y(t)}{y(t)}. \quad (4)$$

$P_h(t)$ is obviously equal to $R_h(t+h)$ (i.e. it is achieved by shifting the returns *h* days backwards). $P_h(t)$ can be interpreted as the profit gained, if buying a stock at day *t* and selling it at day *t*+*h*.

Table 1: Mean *k*-step returns for 207 Swedish stocks

	k						
	1	2	5	10	20	50	100
Mean	0.143	0.274	0.585	1.058	2.007	4.584	8.651
Median	0.000	0.007	0.060	0.248	0.946	2.895	5.148
Std. dev	3.02	4.15	6.15	8.42	11.80	18.82	27.80
Skewness	0.79	1.06	1.02	0.93	0.83	0.78	0.82
Kurtosis	15.78	16.49	11.55	9.27	7.58	5.99	5.59
No of points	1367	1363	1356	1347	1333	1306	1259
Returns=0 (%)	23.4	17.0	10.8	7.5	4.9	2.7	1.9
Returns>0 (%)	38.7	42.0	45.6	48.5	52.1	56.1	57.6
Returns<0 (%)	37.9	41.1	43.6	44.1	43.0	41.2	40.5
Up fraction (%)	50.6	50.6	51.1	52.3	54.7	57.6	58.7
Mean Up	2.7	3.5	5.2	7.1	10.1	16.8	26.5
Mean Down	-2.3	-2.9	-4.0	-5.3	-7.3	-11.3	-15.5

3. Following the Trend

A trend-following trading strategy means buying stocks that have shown a positive trend for the last days, weeks or months. It also suggests selling stocks that have shown a negative trend. In this section the relevance for such a strategy is tested statistically.

In Table 2, the mean profit $P_1(t)$ (Eq. 4) is tabulated as a function of the trend $T_k(t)$ (Eq. 3), i.e. 1-day-forward profit versus k-step-backward trends. Results are presented for the 207 stocks for the years 1987-1996. Table 3 shows the "*Up fraction*" (Eq. 2). Table 4 shows the number of observations in each table entry.

The label for each column is the mid-value of a symmetrical interval. For example, the column labeled **0.00** includes data with the *k*-day trend in the interval [-0.25 0.25[. The intervals for the outermost columns are open ended on one side.

To ensure that found patterns reflect fundamental properties of the process generating the data, and not only idiosyncrasies in the data, the relations between trends and future returns are also presented in graphs, in which one curve represents one year. The left diagram in Figure 1 shows 1-step profits $P_1(t)$ versus 1-step trends $T_1(t)$. The right diagram shows 5-step profit P5(t) versus 5-step trends T5(t).

Figure 1: Profits versus returns for 207 Swedish stocks. Each curve represents one year between 1987 and 1996.

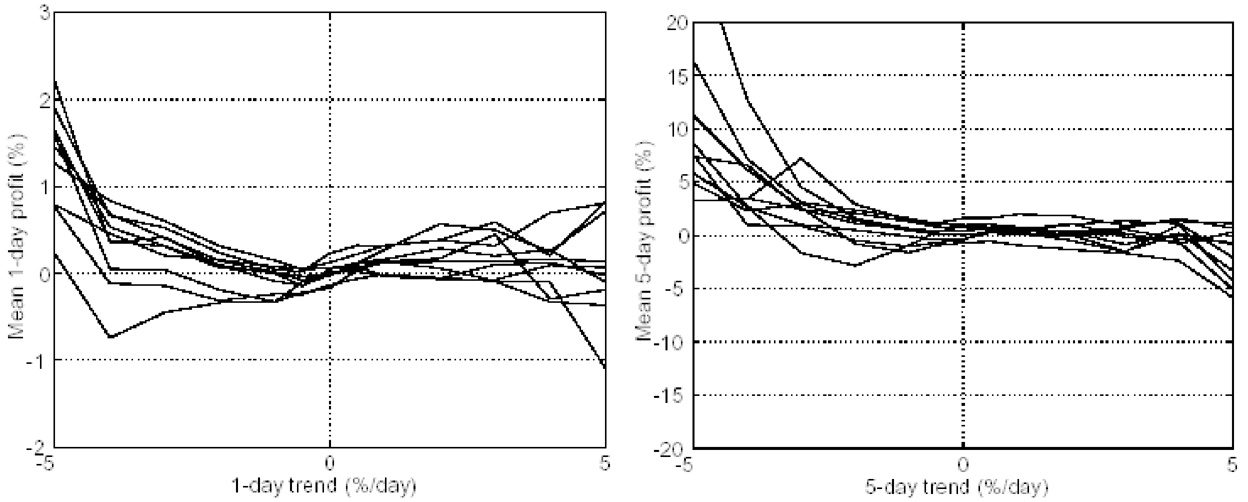

Table 2: Mean 1-day returns

k	k-day trend (%/day)												
k	-5.00	-4.00	-3.00	-2.00	-1.00	-0.50	0.00	0.50	1.00	2.00	3.00	4.00	5.00
1	1.42	0.40	0.30	0.09	-0.02	-0.07	0.02	0.09	0.11	0.14	0.15	0.16	0.06
2	2.35	0.75	0.44	0.22	0.06	-0.00	0.02	0.06	0.09	0.13	0.06	0.10	0.05
3	3.38	0.95	0.62	0.35	0.12	0.01	0.02	0.07	0.11	0.11	-0.00	-0.12	-0.12
4	4.47	1.36	0.65	0.45	0.14	0.04	0.02	0.07	0.12	0.04	-0.00	0.07	-0.03
5	5.06	2.21	0.87	0.40	0.20	0.04	0.04	0.10	0.10	0.05	-0.07	-0.03	-0.01
10	8.28	5.02	1.88	0.57	0.24	0.08	0.06	0.10	0.11	0.13	0.18	0.58	-0.89
20	40.22	11.58	2.43	1.84	0.26	0.05	0.08	0.11	0.16	0.15	-0.10	-0.02	-1.07
30			8.07	3.41	0.38	0.09	0.08	0.10	0.19	0.21	-0.12	0.59	-1.37
50				7.55	0.81	0.08	0.10	0.14	0.11	0.01	0.37	-0.71	-1.05
100					2.51	0.22	0.08	0.15	0.31	-0.15	-0.53	-2.10	-0.30

Table 3: Up fraction (%)

k	k-day trend (%/day)												
k	-5.00	-4.00	-3.00	-2.00	-1.00	-0.50	0.00	0.50	1.00	2.00	3.00	4.00	5.00
1	61.2	56.9	55.0	52.1	49.6	48.0	48.9	50.2	50.0	49.8	49.8	50.0	46.7
2	62.8	57.6	56.9	53.7	51.1	49.5	49.2	49.3	49.8	50.1	48.3	47.5	45.7
3	65.0	58.4	56.5	55.7	51.9	49.4	49.2	49.7	50.1	49.4	47.3	44.6	43.8
4	64.6	58.9	55.5	55.2	52.5	50.1	49.0	49.9	51.0	48.3	46.1	45.8	44.7
5	65.4	60.8	55.7	54.5	52.8	50.0	49.5	50.5	50.2	47.8	44.9	45.5	44.1
10	65.0	65.1	56.0	53.0	51.9	50.6	50.3	50.7	48.9	47.8	46.2	47.9	38.4
20	66.7	65.4	54.8	56.9	50.2	49.5	50.8	50.7	49.1	47.8	43.7	44.0	40.6
30			55.2	57.5	49.4	49.3	51.0	50.4	49.3	47.3	39.8	53.6	37.7
50				57.7	50.5	48.4	50.7	51.0	48.2	46.9	47.9	37.5	41.7
100					52.4	49.1	50.3	50.7	49.3	46.9	47.7	31.6	50.0

Table 4: Number of points

k	k-day trend (%/day)												
k	-5.00	-4.00	-3.00	-2.00	-1.00	-0.50	0.00	0.50	1.00	2.00	3.00	4.00	5.00
1	10256	6255	12158	23991	29276	19134	60845	18447	28262	23213	13091	7077	13141
2	4071	3206	7475	19783	34386	35228	53625	33001	32519	20794	9214	4335	6703
3	2121	2101	5000	15661	33500	42333	58688	39244	32600	18332	6854	2994	4244
4	1187	1426	3640	12431	31945	46192	64660	42539	32581	15915	5432	2264	2982
5	744	990	2852	10142	29791	49017	69243	45101	32426	13989	4384	1698	2344
10	116	255	1047	4837	19894	52570	89411	53333	27421	8062	2238	807	890
20	3	28	168	1941	11475	47613	112230	59258	19290	4344	1029	338	320
30	0	0	32	708	8339	42626	127005	58767	14641	3021	625	209	179
50	0	0	0	59	4033	34769	147072	54372	10147	1655	311	115	83
100	0	0	0	0	364	23997	167490	44255	5972	1094	129	20	53

Let us draw some conclusions from these statistical examinations of trends.

- The massive better part of returns falls into a region, where it is very difficult to claim any correlation between past and future price changes. The regions, where any correlation may be significant, are the sparsely populated extreme ones.
- However, one interesting e.ect can be observed. Looking at Table 3, we observe that a 5% decrease in price (or more precisely: a return < -4.5%) since the previous day, stands a 61.2% probability of showing an increase by the following day. This effect justifies the notion of a *mean reverting effect* commonly used in technical analysis. The effect is confirmed in the yearly analysis presented in Figure 1. The mean reverting effect after a large drawdown is present both at 1-day and at 5-days prediction horizon. It is also clear that no corresponding conclusion regarding the effects of large increases can be drawn. For these cases, future returns are randomly distributed around zero, both at 1-day and at 5-days horizon.

4. Day-of-the-Week Effect

In this section we investigate how the day of the week affect the stock price returns. The results confirm and complement similar investigations on other stock markets world-wide. The day-of-the-week effect has been studied in a number of research papers. Hawawini and Keim [2] present a summary, which demonstrates significant differences in average daily returns across days of the week. In our investigation of the Swedish stock market, the daily returns are presented in a somewhat different fashion than is normally done. The stock returns are computed for all twenty-five combinations of buy and sell days. The returns are presented as "daily returns", i.e. they are divided by the number of calendar days between buy and sell. For example, the return from buying on Friday and selling on Monday is divided by three before it is put in the table. A second table with the same layout presents the "*Up Fraction*" (Eq. 2) for the same combinations of buy and sell days. In this way, all combinations of buy and sell days can be compared on an equal basis.

As before, results are presented for the 207 stocks from the Swedish stock market for the period 1987-1996. This provides statistically more stable grounds than using one single index (e.g. [2]). The daily returns R are shown in Table 5 and the "*Up Fraction*" in Table 6. We can extract several interesting "anomalies" from these tables:

- The day-of-the-week affects the returns significantly. The returns span between 0.003% (buy Friday/sell Tuesday) and 0.243% (buy Thursday/sell Friday).
- The one-day returns increase monotonically from Monday to Thursday: 0.004, 0.114, 0.167, 0.243 (buying on a Friday never yields a one-day return).
- The right most column describes the mean returns achieved when selling between one and seven days from the buying day. Friday and Monday appear to be the worst days to buy in this 1-day perspective.
- Looking at "*Up Fraction*", it is still clear that the real trading odds are almost as bad as before. Even if we pick the best choice and buy on Thursday and sell on Friday, we loose money in 47.96% of the cases. It would take great patience and a stable financial backup to utilise the shown day-of-the-week effect.

A question that should be posed always when looking for and finding structures in huge data sets, is whether the found structure reflects some general property of the data generating process, or is simply an effect of data snooping. In this particular case, we have calculated the same statistics for yearly data over 1987-1996. In this way, the results are tested for stability in time. The reported effects are present even in these cases, and thus provide additional support for the results. However, the risk for data snooping is, as always in the case with stock data, huge.

Table 5: Daily returns (%) for combinations of Buy and Sell days

	Sell Day					
Buy Day	Mon	Tue	Wed	Thu	Fri	Mean
Mon	0.069	0.004	0.050	0.078	0.105	0.061
Tue	0.081	0.066	0.114	0.136	0.152	0.110
Wed	0.076	0.060	0.066	0.167	0.186	0.111
Thu	0.061	0.044	0.051	0.058	0.243	0.091
Fri	0.014	0.003	0.018	0.033	0.062	0.026
Mean	0.060	0.035	0.060	0.095	0.150	0.080

Table 6: Up fraction (%) for combinations of Buy and Sell days

	Sell Day					
Buy Day	Mon	Tue	Wed	Thu	Fri	Mean
Mon	50.21	48.42	49.07	49.89	50.66	49.65
Tue	50.85	50.68	50.39	51.24	52.04	51.04
Wed	50.79	50.27	51.16	51.63	52.40	51.25
Thu	49.80	49.38	49.86	50.41	52.05	50.30
Fri	48.70	47.51	48.18	49.02	50.30	48.74
Mean	50.07	49.25	49.73	50.44	51.49	50.20

Table 7: Number of observations for combinations of Buy and Sell days

	Sell Day					
Buy Day	Mon	Tue	Wed	Thu	Fri	Mean
Mon	50605	54476	53933	52653	51495	52632
Tue	54044	56810	57756	56206	54845	55932
Wed	53794	57100	56546	56235	54826	55700
Thu	52634	55865	55880	54341	54260	54596
Fri	52898	54781	54790	53798	52454	53744
Mean	52795	55806	55781	54647	53576	54521

5. Month Effects

The month effect on stock returns is investigated by computing daily returns for each month. The returns are computed for the years 1987-1996. The mean results for the 207 stocks are shown in Table 8. These results for the Swedish stock market fit well with investigations on other markets. Hawawini and Keim [2] present a summary of a research on a number of stock markets worldwide. The high returns for January and low returns for September are significant for most of the markets, including the Swedish stock market. The Up Fraction varies between 47.24% (August) and 53.20% (January). The mean Up Fraction is 49.94%, which is close to the 50%, proposed by the random-walk hypothesis. Note, that a prediction accuracy of about 54% hit rate for the sign is often reported for elaborate prediction algorithms. Most algorithms do not use any calendar data as input variables, see example. [4] or [1], and claim to show predictive capability in the algorithms. Be that as it may, if we can achieve a similar hit rate by just looking at what month we are trading in, it seems reasonable to incorporate in some way the month-of-the-year in the algorithm. And the validation process really should be reconsidered for algorithms that do not do that.

5.1 Monthly Returns for Combinations of Buy and Sell Months

We conclude the investigations of seasonal effects with a trading-oriented statistical test, where both buying and selling are considered. The first trading day in each month is always selected for both buying and selling. After buying in the beginning of a month, the returns from selling in the beginning of each of the successive twelve months are computed and stored in a twelve-by-twelve table. The shown figures in Table 9 are daily returns times 30, to obtain comparable monthly returns for all months, regardless of the number of days they contain. Table 10 shows the *Up Fraction* (Eq. 2) for the same combinations of buy and sell months. The right-most column shows average values for each month. We can conclude that December, January and February are good months to buy stocks, whereas August and September produce the lowest profits in average over the investigated period. It is important to realise that the presented figures are average values that are very sensitive to the market's behaviour during individual years. A more detailed yearly analysis [3] shows that the spread between years is considerable.

6. Summary

We sum up the results with some of the most interesting observations.

- The analysis of trends show very weak support for a general trend concept for the stock market. The massive better part of returns falls into regions, where it is very difficult to claim any simple correlation between past and future price changes. A possible effect is the mean reverting behaviour: a 5% decrease in price since the previous day, stands a 61.2% probability of showing an increase by the following day. The cases with large *increase* since the previous day exhibit no similar effect.
- The presented statistics show significant day-of-the-week and month-of-the-year effects on the stock returns. The daily returns vary between 0.004% (buy Monday/sell Tuesday) and 0.243% (buy Thursday/sell Friday).

  The *Up Fraction* was shown to depend on the month of the year and to vary between 47.24% (August) and 53.20% (January). Even if the effects are too small to be utilized in actual trading, they are definitely big enough to influence other prediction algorithms, such as ordinary time series analysis or neural network models of daily returns. If not taken into account in such algorithms, the seasonal effects appear as a high noise levels in the data. It was shown, that the month-of-the-year effects are of the same size as the accuracy of many published prediction algorithms that do not make use of any date information.

There are several ways to deal with the calendar effects when constructing prediction algorithms:

- Include the time dimension in the modeling, i.e. include a trainable parameter describing how the return depends on the day of the week, or on the month.
- Aggregate data. For example, instead of modeling the return time series for all the days in a given time period, we can restrict the model to predict from one Monday to the next.

Table 8: Average daily returns for each month

	Mean	Std.dev.	Incr.fraction	No. of obs.
Jan	0.357	3.90	53.20	24058
Feb	0.208	4.03	50.76	23590
Mar	-0.025	3.43	47.35	26661
Apr	0.202	3.28	52.67	23408
May	0.152	3.47	51.62	24406
Jun	-0.026	3.23	48.10	24914
Jul	0.220	3.09	52.99	26042
Aug	-0.096	3.45	47.24	27941
Sep	-0.042	3.78	48.49	27338
Oct	0.036	4.57	48.77	28515
Nov	0.082	4.04	48.91	27764
Dec	0.111	4.56	50.33	26447
Mean	0.092	3.78	49.94	25924

Table 9: Monthly returns (%) for combinations of buy and sell months

	Sell Month												
Buy Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Mean
Jan	1.47	6.36	4.79	2.70	2.91	2.92	2.35	2.70	2.18	1.91	1.87	1.76	2.83
Feb	1.14	1.91	3.91	1.34	1.95	2.27	1.76	2.40	1.82	1.65	1.61	1.49	1.94
Mar	0.52	1.27	1.33	-0.58	1.12	1.60	1.10	1.77	1.22	1.01	1.00	0.94	1.03
Apr	0.73	1.52	1.51	1.12	3.31	2.80	1.70	2.46	1.59	1.27	1.23	1.13	1.70
May	0.35	1.25	1.27	0.87	1.18	2.56	0.90	2.07	1.09	0.78	0.82	0.73	1.16
Jun	-0.06	0.89	0.99	0.64	0.97	1.08	-0.65	1.49	0.51	0.27	0.26	0.29	0.56
Jul	0.16	1.20	1.32	0.85	1.19	1.28	1.13	4.01	1.11	0.62	0.57	0.54	1.16
Aug	-0.82	0.43	0.63	0.28	0.66	0.85	0.72	1.01	-1.53	-1.07	-0.68	-0.32	0.01
Sep	-0.49	1.06	1.24	0.76	1.13	1.31	1.15	1.54	1.28	-1.04	-0.45	0.08	0.63
Oct	-0.05	1.82	2.00	1.29	1.61	1.81	1.57	2.06	1.67	1.45	0.15	0.74	1.34
Nov	0.07	2.32	2.43	1.58	1.86	2.10	1.80	2.26	1.85	1.60	1.82	1.25	1.75
Dec	0.35	3.68	3.38	2.10	2.34	2.51	2.09	2.48	1.97	1.70	1.90	1.85	2.20
Mean	0.28	1.98	2.07	1.08	1.69	1.92	1.30	2.19	1.23	0.85	0.84	0.87	1.36

Table 10: Increase fraction (%) for combinations of buy and sell months

Buy Month	Sell Month: Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Mean
Jan	53.89	65.12	66.08	59.48	66.08	66.28	64.06	66.46	61.84	62.26	59.27	59.68	62.54
Feb	50.37	52.39	55.03	53.02	58.68	59.53	59.04	61.10	57.50	57.52	55.35	54.84	56.20
Mar	46.57	48.89	50.81	43.03	54.95	58.05	56.05	58.75	54.41	55.24	51.90	50.84	52.46
Apr	49.61	51.78	53.77	52.19	63.64	67.02	60.71	65.54	59.84	58.24	54.95	53.48	57.56
May	45.23	48.54	50.43	48.10	52.02	56.57	53.15	58.53	55.26	54.60	52.05	49.93	52.03
Jun	44.73	48.84	49.83	47.32	51.74	54.28	45.88	55.58	50.76	53.60	49.64	48.16	50.03
Jul	47.06	51.54	52.45	49.58	54.59	57.49	56.92	60.93	52.15	54.39	51.18	50.92	53.27
Aug	41.08	47.54	47.13	44.33	48.56	52.13	52.52	53.74	41.95	46.21	45.68	45.70	47.21
Sep	44.80	52.16	53.26	50.37	53.95	57.26	56.74	59.05	54.60	49.64	47.31	48.80	52.33
Oct	46.50	56.02	56.83	53.48	56.41	58.62	58.81	58.36	55.75	55.22	47.56	50.53	54.51
Nov	44.69	61.16	60.67	55.06	61.18	61.67	60.56	61.15	56.70	57.31	55.61	48.80	57.05
Dec	49.87	65.49	66.11	59.71	64.90	66.00	64.49	66.26	60.43	61.12	59.27	61.07	62.06
Mean	47.03	54.12	55.20	51.31	57.23	59.58	57.41	60.46	55.10	55.45	52.48	51.90	54.77

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April 19, 2002

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