A Lative Logic view of the Filioque Addition

Patrik Eklund

Umeå University, Department of Computing Science, SE-90187 Umeå, Sweden
peklund@cs.umu.se

1 Properties of logical operators

Logic and natural language are comparable to the point that they both need rules for formation of words and sentences, and that words and sentences are in some sense expected to be lative with respect to one another. Further the formation rules are different for words and sentences, and words appear within sentences. For natural language this view can be formalized to some extent, and for logic the situation can be analyzed more mathematically. In such an analysis, however, we need to distinguish whenever we aim at 'logic for mathematics' or 'mathematics for logic'. Logic as developed during late 19th century indeed, as Hilbert often pointed out, developed hand in hand in axiomatic set theory. First-order logic needed within axiomatic set theory is, as a logic, not the same as defined e.g. within [10, 6], where category theory is the metalanguage for definition and analysis of logical notions. Category in turn, as an object language, uses axiomatic set theory as a metalanguage.

Properties of logical operators are also often taken for granted, in particular when facts are bivalent. Commutativity of conjunction and disjunction is a typical example where we usually do not question this property. However, when we look at an example like "from x and_from y" and "from y and_from x", it is clear they are not necessarily the same. A syntactic expression like "from (x and y)" could be treated as the same as "from (x and y)" if "and" is understood as commutative. Obviously, "from" in "from(...)" is blind for "...". An expression like "from x and y" is tricky if we do not recognize the parenthesis. This shows how non-commutative subordinations may be treated as being commutative. The latin 'nisi' for 'unless' is also interesting in a predicate like $Q(x, y)$ for "knowing x unless [knowing] y". Obviously, Q is not eo ipso commutative, so if in some context we want both to hold, i.e., $Q(x, y)$ and $Q(y, x)$ at the same time, we need to do add both explicitly. St. Matthew 11:27 states that "no man knoweth the Son, but the Father; neither knoweth any man the Father, save the Son" (nemo novit Filium nisi Pater neque Patrem quis novit nisi Filius). "Gloria Patri, et Filio, et Spiritui Sancto", but 'et' is non-commutative, even if "three is one" in Christian Triadology.

Within the square of opposition, "some S is P" is commutative if written in first-order logic as $\exists x.Sx\&Px$. This rewriting may therefore not be appropriate, and first-order logic may be not expressive enough as a language for explaining the square in its various forms.

2   Distinction between term and sentence

Lativity in logic, and thereby aims to be careful about self-referentiality, is then at least of two-fold. On the one hand, metalanguage and object language should not be mixed, and if there is a chain of languages one being meta to the other, it is not advisable to close the loop. On other hand, within a language we indeed have a lativity between words and sentences, and for logic languages, also between truth and provability, the latter building upon proof rules. In logic, we should be lative, respectively, when dealing with signatures, terms, sentences, structured conglomerates of sentences, models, entailment, axioms and inference rules. The role of signatures, i.e., sorts and operators, is fundamental for sortedness aspects. Unsorted views, like appearing in the "fons et origo" first-order logic and axiomatic set theory easily allow for "mixed bags" in particular when dealing with terms and sentences, but also when mixing truth and provability. A typical appearance of this 'mixing bags' situation appears in Gödel's numbering [5]. Using notations in [8], a predicate symbol A and a predicate $A(x)$ invites to speak about "$A(x)$ is provable" and using the notation "$\vdash A(x)$". However, proceeding to create a "metamathematical proposition" $\mathfrak{R}(x, Y)$, representing "Y is a proof of $A(x)$", then allowing to write

$$(\exists Y)\mathfrak{R}(x, Y) \ \equiv \vdash A(x)$$

and at the same time wondering "What is the nature of the predicate $\mathfrak{R}(x, Y)$?", requires a by-passing by saying it must be an "effectively decidable" metamathematical predicate, and that "there must be a decision procedure or algorithm for the question whether $\mathfrak{R}(x, Y)$ holds". Mathematical propositions and metamathematical propositions are thus allowed to be in the same bag, and in [5] there is frequent use of that degree of freedom to mix bags.

Concerning words and sentences it is also important to note the classical typing problem, in particular related to propositions, as pointed out already by Church [2] concerning the distinction between the "sort of the sorts of terms" from the "sort of sentences". This was implicitly observed by Schönfinkel [13] in his unsorted approach. Concerning words, and indeed terms in logic languages, underlying signatures involve type constructors also sensitive to invite a mixing of bags. Simple type theory and λ-calculus is careless not to identify levels of signature, as has been pointed out in [4].

The use of terms in sentences is more transparent e.g. in equational logic, but in first-order logic it is not so. Indeed, not clearly making distinctions between term and sentence and when a symbol really acts as an operator or just as a selector or pointer often leads to constructions which are harder to defend. In [4] we showed how we can use levels of signatures to provide a precise term functor, extendable to a monad in order to allow for composable substitutions, for constructing λ-terms. The key point is that 'λ' should not be seen as a universal abstractor of operators, as also pointed out in [2], e.g. in expressions like $\lambda x.\omega(x)$, but indeed that "ω owns its abstraction". In this way the problem of renaming can be avoided [4]. In other words, en external λ is not the one

to point out the place in the domain of ω where x resides, but ω does it for itself. Similarly we may look at the existential quantification and expressions like $\exists x.P(x)$ as compared with $P(x)$. In traditional first-order logic they are considered as statements or predicates, but strictly speaking P is an operator symbol in some signature, so that $P(x)$ is a term, not a sentence. A negation operator $\neg$ can be applied to the term $P(x)$ so that $\neg P(x)$ and $P(x)$ are of the same sort, as terms. However, $\exists x.P(x)$ is not a term, but intended to be used as a sentence, and it must be questionable whether $\neg$ in $\neg\exists x.P(x)$ and $\exists x.\neg P(x)$ really can be the same symbol. In $\exists x.\neg P(x)$, it acts as an operator, changing a term to a term, but in $\neg\exists x.P(x)$ it changes a sentence to a sentence, so it is strictly speaking not an 'operator'. An important distinction between terms and sentences is also that terms are constructed by term monads, allowing composable substitutions, whereas sentences are formed using sentence functors, not necessarily extendable to monads, i.e., not enabling propositional variables and substitutions. First-order logic as a "fons et origo logic" and restricted to be used in axiomatic set theory is clear and transparent for its purpose, not as foundations for mathematics but as a metalanguage for set theory, but looking at $\exists x$ objectively with category theory as metalanguage, similarly as we may investigate λx, reveals how this "operability" is very suspicious.

Self-referentiality is more in existence than in being, and philosophy and philosophical logic during the latest centuries focuses quite well on being and existences, but on the other hand also takes more liberties concerning self-referentiality. The distinction between being and existence is hard to describe logically, in particular in some logic adopting an unsorted approach. Having no typing and no formal distinction between terms and sentences allows not just for self-referentiality, but also sentence constructions that implicitly mixes sorts. This mixing of sorts appears e.g. in [12] using the "collection of axiomatizations" basically as an index set, but not even as an index set of sorts but as a way of indexing logics. Creating a sentence (in an equational style logic) like $S_{g_i}(x_{g_i} + y_{g_i}) = S_{g_j}(x_{g_j} + y_{g_j})$ then takes the terms $S_{g_i}(x_{g_i} + y_{g_i})$ and $S_{g_j}(x_{g_j} + y_{g_j})$ from different logics and creates a sentences in a common logic for which their is not necessarily a counterpart in the "collection of axiomatizations". Generalized general logics [4, 7] aims to explain this phenomenon of mixing bags and indeed provides a lative view of logic language constructions, where signatures, terms, sentences, axioms, entailments and proof rules are all kept strictly apart from each other.

Logic is in the end mostly about 'true' and 'false', and this is different from 'right' and 'wrong', in particular from the viewpoint how one is antithetic to the other, false to true, and wrong to right. Intuitively speaking, the logic of 'true' and 'false' is hardly identical to the logic of 'right' and 'wrong', where the former intertwines perhaps more with reason, attitude and law, whereas the latter more with ethics, behaviour and faith. Analyzing language and translations, either natural or logical, in these contexts clearly requires attention to language constructions and meaning.

3 Logical aspects of the Filioque Addition

In the beginning was the Word (St. John 1:1), and it might be tempting to say that there is a distinction to be made between the Spirit being in Faith and Logos being in Law. According to the Creed, with the "Filioque" issue as a difference between the Latin and Greek Churches, the Spirit "proceeds from the Father through the Son", where the role of the Son with respect to being causative or mediative has been debated. In Christianity, the role and notion of Church may also invite to say that Logos, as written in language, proceeds through the Church, includes Sacraments, and embraces teaching of Faith. This proceeding of Spirit and Logos, respectively, through Son and Church, is formulated in language, passes over time through translations and controversies, and ends up in different formulations.

We have used and we still use (natural and native) language to speak and write about the Word. However, we should not abuse language to speak and write about the Word. Can Language 'explain' the Word, or are writings written in Language just written representations of the Word? Is there a "correct and complete" way to explain and/or write? There are canonical writings, but is there a canonical way to write about these writings? There is perhaps an ecumenic way to write about the writings of the writings, but not an ecumenic way to write about the writings? This changes over times, as later ecumenic councils look backwards, affirming, or not affirming, what is and what isn't. The ecumenic councils 869-870 and 879-880 were critical, and not just because of 'Filioque'.

The Arian controversy, which Augustinus fought against intensively, comes to a culmination in the Toledo council 589, where the Spanish Church stood up against the Arianist Visigoths. The Filioque Addition in the Creed appears centuries before that, and the role of the Son in proceeding of the Holy Spirit from the Father came to be a debate instrumented by Augustinus in his fight against Arianism [11]. The Filioque Addition adopted by the Latin Church is since then never accepted by the Greek Church, and the Byzantine triadology indeed still rejects any causative participation from the Son in the proceeding of the Spirit. From logic language point of view, it is not immediately defendable to read "Patre Filioque" as "Father and Son", where 'and' is conjunctive, but indeed reading "qui ex Patre Filioque procedit" as "comes from the Father and proceeds through the Son". This removes the 'and' as a conjunction between 'Father' and 'Son', since the reading is then more equivalent to "coming from the Father, [and] proceeding through the Son", and from that the Trinity would be explained as if the Spirit isn't 'given' until through the Son (St. John 7:39). It looks more like a sequence, or continuation, or even a thread. Viewing it as a 'thread' is obviously because fil- and filum can be interpreted more as a thread, whereas "fili-" as well as "filius" and "filia" is "son" and "daughter". This then would dissect the clauses in some additional ways, encapsulating and explaining the being and not being of that conjunctive 'and'. The Greek "ekporeuomenon" should clearly be seen in relation with the Aramaic "npq" and the the Latin "procedit", e.g. as in St. John 15:26. However, they may be slightly different in respective languages, but that is indeed how it was translated at that time. Clearly, efforts

to translate sentences expressed in natural languages like Aramaic, Greek and Latin to corresponding sentences in logic oriented languages, must then respect both syntactic as well as semantic aspects. Further, the latin Church indeed says "qui ex Patre Filioque procedit", but not "qui ex Filio Patreque procedit", since "Filio Patreque" makes no sense Clearly, `-que` is a non-commutative (logical) connective.

The logical understanding of "Filioque" surely must be understood, at least partly, by Augustinus' arguing against Arianism and their "procedit a solo Filio", where Augustinus says "nec a solo Filio missum est, sed a Patre quoque". Whether or not the conjunction 'and' should be placed between 'Father' and 'Son', and indeed how to understand respective causative and mediative content in "proceeding from the Father through the Son", cannot be solved by logic, but the different formulations caused by natural language constructions and strategic argumentation can be enlightened by discussing logic language counterparts. Obviously, the target for Augustinus in this dispute was "nec a solo", and thereby the discussion on cause and mediation became less visible.

4 Meaning change in translation

We may also observe how meaning change in translation, taking St. Matthew 6:6 as an example. As pointed out implicitly by Augustinus in his *De Sermone Domini In Monte* (394), Jerome did the modification from "and while you close the door" (Vetus Latina: claudentes ostia) to "and when thou hast shut thy door" (Vulgata: clauso ostio). The reason for this change may be mostly unknown but one may speculate that the reason for this change is liturgic, since a ceremony is always "sequential" in some sense, i.e., in the style of "first do this, and then this, and then that". As pointed out in [11], "Filioque" was stated already at that time in the shift from Vetus Latina to Vulgate. Therefore it may not be clear that we can justify the understanding of that Filioque Addition only from the viewpoint of reacting to the Arian statements, as Augustinus did, or if the Filioque addition also was aiming at some more simple and clear phrasing from liturgic point of view. Is it perhaps so that we may not need not be so much ecumenic about the "Spirit proceeding through", since there is actually no real dispute in this matter, as compared to what we need to be about the "Logos proceeding through"?

Various other attempts appear which logically try explain existence and Trinity. These efforts play with predicates and existence symbols in the Aristotelian tradition and in quite a careless way, which easily leads to "mathematically heretic" formulations e.g. as seen in Sabellius' Arguments or efforts to encode the Shield of the Trinity in simple first-order logic [9].

Translations related to revelation are also somewhat different if we look at "What we [can] know [about God and goodness] is what has been revealed [to us]" (Romans 1:19).

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quia quod notum est Dei manifestum est in illis Deus enim illis
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manifestavit (Vulgata)
Denn was man von Gott weiß, ist ihnen offenbar; denn Gott hat es
ihnen offenbart, (Luther 1545)
Because that which may be known of God is manifest in them;
for God hath shewed it unto them. (King James Version 1611)
Vad man kan känna om Gud är nämligen uppenbart bland dem;
Gud har ju uppenbarat det för dem. (Swedish 1917)
sentähden että se, mikä Jumalasta voidaan tietää, on ilmeistä
heidän keskuudessaan; sillä Jumala on sen heille ilmoittanut.
(Finnish 1933/38)
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5 Communication and dialogue

In divine revelation, "dialogue" is of different type than "dialogue" between humans. Thomas Aquinas' in his Compendium refers to Augustine's commentary on John, and states "no one knows God unless He who knows manifests Himself" (Deum nullus cognoscit, si non se indicat ipse qui novit).

The question of will and free will in "God reveals will" also a question about state or moment. State of will and moment of will may not be the same. Propositional revelations are truths revealed by God but they are not verified using human reason.

Concerning the Latin 'loqui' for 'to speak' and 'locutio' for 'speech', it is important to note that in interior locution, something is spoken, or something is "delivered", or something "is just there", exists, or simply "is". Should we then say "it is a dialogue", or would we just say "it is"?

6 Logic through history

Can Natural Language explain Logic? Can Logic explain Natural Language? Concerning "Language (structure) and Word", and "Language (structure) and Church", is that the same "Language (structure)"? Is the related "Language and Logic" the same? Maybe it is so that logic and reason can be enriched by Something, or Spirituque, that is in the Word and which proceeds through Natural and Native Language? Even if this is transcendental to scientific approaches and logic language, there seems to be soething to learn from the way theological writings have evolved through history.

Whereas Augustinus didn't debate Aristotelian logic, this debate opens up centuries later when Thomas Aquinas rightfully points out that Aristotle didn't try to explain the difference between being and existence. Clearly, Aristotle was not affected by the Logos as Thomas Aquinas was, but Aristotle saw the potential danger in using self-referentiality. Aristotle, in his *Prior Analytics*, said "a true conclusion may come through what is false", and he also speaks about "the same terms", and then the question is what he means by a "term". Saying "positive terms in positive syllogisms" indicates that terms are sentences, but then "positive" may have at least two different meanings. In his statement "it is

impossible that the same thing should be necessitated by the being and by the not-being of the same thing", Aristotle then mixes truth and provability, and trying to make that into a "sentence". Aristotle's statement "just as if it were proved through three terms" clearly reveals how Aristotle becomes intertwined since he does not separate truth from provability. We could say that Aristotle was very "illative" and clearly he was very "unsorted".

Thomas Aquinas identifies 'being' as something needed before 'existing'. Univocality and equivocality concerning *names* becomes important in his *Summa Theologiae*, and these names are implicitly said to respect sorts, or at least, names are comparable if various things attributed to these names are comparable. When we say that "things attributed are univocal" we are logically closer to dealing with terms, and in fact vocality then relates to comparativeness and transformation within logical many-sortedness. However, for a unary predicate symbol P (like barking) and a variable x (e.g. for dogs), with that variable as a term being of the same sort as the domain sort of the operator P, we must be quite precise about the meaning of univocality between $P(x)$ (barking dog) and x (dog). Similarly, if P would denote something like 'going out of' and x something like 'that which', then 'procedit' and 'npq' are closer to being names of a predicate or operation symbol, whereas 'ekporeuomenon' expresses the sentence or term. It would be far fetched to say that Thomas Aquinas is the starting point for type theory and many-sortedness in logic, but he certainly touches upon the issue of vocality in the sense of sortedness.

Analogy, also an important theme in Thomas Aquinas' writings, on the other hand, is not just comparing and matching sorts, but rather a transformation of one sort to another, as a naming basis for transformations between Worlds. For Logos we should say that there is one World, but we could also say that there may be differently perceived Laws, since they are received in different Worlds. Analogy is, category theoretically speaking, based on signature morphisms when providing analogy between terms [4], and natural transformations between sentence functors when dealing with predication.

The Thomism view on the difference of being and existence is discussed also in [1], where essence is symbolically introduced by 'E' and the "essence of x", symbolically written Ex, is required to be an ingredient in the "existence of x, written $\exists x$, which is seen as an operator producing either existence $\exists x$ or non-existence, denoted $\sim \exists x$. This is done given E and an x appearing in some "universal collection" V, where that universe is not further explained or defined, nor is it sorted in any way. If the passage from Ex to $\exists x$ actually takes place, an external flag is raised ($c = 1$), otherwise not ($c = 0$). The symbol $\sim$ is bound to that 'taking place', and indeed to the 'value' of c, but they remain external or reside in some metalanguage not explicitly explained or presented. The equality symbol in [1] is explicitly made into a metalevel symbol, but appears in sentence formulations in a similar ways as compared with the treatment in [12].

Concerning existence in general, interesting is also that 'existence' really means 'being out of', i.e. "ex sistere", as frequently pointed out e.g. by Heidegger in his *Sein und Zeit* (1927). Logicians after Frege have not acknowledged this

phenomenon well enough, and 20th century logicians often felt inclined to present their relations with Principia Mathematica before trying to provide new innovations in logic. Surprisingly little came out in the 1950's and 1960's given novel ideas emerging in Göttingen and Vienna, and from the Lvov-Warsaw School. Within the efforts to use logic language in theology, the Cracow Circle provided some contributions, but remained in the shadow e.g. of the Lvov-Warsaw School.

Moving from Thomas Aquinas' time a few centuries forward, theatre and drama again becomes an important part of society. Shakespeare's literary part can be seen as more logical and factual, whereas the theatrical part including drama is more about process and progression [3]. Shakespeare's theoretical thinking happens during late 16th century and early 17th century, so the style and maturity of this "logic" can perhaps be understood e.g. from writings of Descartes and Pascal. It is surprisingly seldom pointed out that Descartes' "cogito ergo sum" was preceeded by Augustinus' "si enim fallor, sum" (if I fail, I am) in his *De civitate Dei*. Pascal on the other hand said we should not define 'being' by 'it is', and thereby he clearly disagrees with Descartes' "cogito ergo sum". Pascal's *De l'esprit géométrique et de l'art de persuader* speaks warmly e.g. about using only nominal definitions, and being very careful never to use the same name or notation for two different things. He also pointed out the need of avoiding self-referentiality, and he used e.g. the example "motus nec simpliciter actus nec mera potentia est, sed actus entis in potentia" (motion in not just action, nor pure force, but action being in force). De Sacy grounded his discussions with Pascal by leaning more towards theology and Augustinus writings than on purely philosophical statements. Some months before the discussions with De Sacy, Pascal had his religious visions which affected his writings during the last years of his life. The *Port-Royal logic* was first published in 1662, the year Pascal died, and Pascal is believed to have produced significant parts for that book, that remained as a popular textbook in logic throughout the pre-fregean time. The propositional attitude, explicitly pointing at the relation between a person and a proposition, adopted by the Port-Royal logic was then more or less totally abandoned by post-fregean logic. Pascals view on typing can be seen from his statement that a cluster of houses makes a village, but a single house is not a non-village. There is a significant different between a 'thing' being, and a 'non-thing' being. Pascal in saying that non-divisible does create eternal space the also supports this claim by saying that something non-divisible and eternal space is of different type.

Thus we may say that throughout the 19th century history of logic there has been many moments of opportunities to maintain and further develop logical innovations initiated e.g. by Thomas Aquinas, Blaise Pascal and John Stuart Mill, but surprisingly little prevail, and modern logic seems to focus more on what can be computed rather than what can be explained.

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