# Semantic Argumentation using Rewriting Systems

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In this article, we introduce a general framework for *structured argumentation* providing consistent and well-defined *justification* for conclusions that *can* and *cannot* be inferred and there is certainty about them, which we call semantic and NAF-arguments, respectively. We propose the so-called *semantic argumentation* guaranteeing well-known principles for quality in structured argumentation, with the ability to generate semantic and NAF-arguments, those where the *conclusion* atoms are semantically interpreted as *true*, and those where the conclusion is assumed to be *false*. This framework is defined on the set of all logic programs in terms of *rewriting systems* based on a *confluent set* of *transformation rules*, the so-called *Confluent Logic Programming Systems*, making this approach a general framework. We implement our framework named *semantic argumentation solver* available open source.

# 1 Motivation

We propose a general argument construction based on the partial interpretation of programs using different *families* of *logic programming semantics* induced by *rewriting systems functions* [\[6\]](#page-3-1). *Rewriting rules* are used to replace parts of a logic program based on the concept of a *normal form*, which is the least expression of a program that cannot be rewritten any further [\[9\]](#page-3-2). For example, having a program with the only rule: *innocent*( $x$ )  $\leftarrow$  *not guilty*( $x$ ), current structured argumentation approaches [\[10\]](#page-3-3) generate the only consistent argument:  $\langle \{ \text{innocent}(X) \leftarrow \text{not guilty}(x) \}, \text{innocent}(x) \rangle$ , expressing that person *x* 

| {z } *Support* | {z } *Conclusion*

is innocent if *x* can not be proved guilty. However, in domain applications that need the generation of argument-based *reason explanations*, providing structured and well-defined reasons why *x* is not guilty (*not guilty* $(x)$ ) are needed. We emphasize the role of investigating such computational mechanisms that can also build arguments justifying conclusions based on the atoms that are inferred as *false*, *e.g.*, to state that *there is certainty in affirming that the guiltiness of x is false (there is no evidence), therefore the x must be innocent, i.e.,*  $\{\{\text{innocent}(X) \leftarrow \text{not guilty}(x)\}, \text{not guilty}(x)\}.$  These types of arguments *Support* | {z } *Conclusion*

have been less explored in the formal argumentation theory, except for *assumption-based argumentation* (ABA) [\[8\]](#page-3-4).

# 2 Syntax and semantics

We use propositional logic with the following connectives  $\land, \leftarrow, not$ , and  $\top$  where  $\land$ , and  $\leftarrow$  are 2-place connectives, *not* and ⊤. The negation symbol *not* is regarded as the so-called *negation as failure* (NAF).

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© E. Guerrero, J.C. Nieves This work is licensed under the [Creative Commons](https://creativecommons.org) [Attribution](https://creativecommons.org/licenses/by/4.0/) License. We follow standard logic programming syntax, *e.g.*, [\[5\]](#page-3-5), for lack of space we do not include some basic and well-established syntax.

An *interpretation* of the signature  $\mathcal{L}_P$  is a function from  $\mathcal{L}_P$  to {false,true}. A *partial interpretation* of  $\mathscr{L}_P$ , are the sets  $\langle I_1, I_2 \rangle$  where  $I_1 \cup I_2 \subseteq \mathscr{L}_P$ . We use SEM $(P) = \langle P^{true}, P^{false} \rangle$ , where  $P^{true} := \{ p | p \leftarrow p \}$  $\top \in P$ } and  $P^{false} := \{p | p \in \mathcal{L}_P \setminus \text{HEAD}(P)\}\$ . SEM(*P*) is also called model of *P* [\[6\]](#page-3-1). We use three value semantics that are characterized by *rewriting systems* following a set of Basic Transformation Rules for Rewriting Systems (see details in [\[6\]](#page-3-1)), those rules are named: RED+, RED-, Success, Failure, **Loop, SUB, and TAUT.** Then, two rewriting systems ( $\mathcal{CF}$ ) can be defined based on the previous basic transformations:  $CS_0 = \{RED^+, RED^-, Success, Failure, Loop\}$ , induces the WFS [\[3\]](#page-3-6).  $CS_1 = CS_0 \cup$ {*SUB*,*TAUT*,*LC*}, induces WFS+ [\[6\]](#page-3-1). The *normal form* of a normal logic program *P* with respect to a rewriting system  $\mathscr{CF}$  is denoted by *norm* $\mathscr{CF}$ . Every rewriting system  $\mathscr{CF}$  induces a 3-valued logic semantics  $SEM_{\mathscr{C}}\mathscr{S}$  as  $SEM_{\mathscr{C}}\mathscr{S}(P) := SEM(norm_{\mathscr{C}}\mathscr{S}(P))$ . To simplify the presentation, we use the entailment  $\models_{\text{SEM}_{\mathscr{C},\mathscr{S}}}$  applied to a logic program *P* is defined by SEM $_{\mathscr{C},\mathscr{S}}(P) = \langle T,F\rangle$  in which  $P \models_{\text{SEM}_{\mathscr{C},\mathscr{S}}} a$ if and only if  $a \in T$ , similarly, if  $P \models_{\text{SEM}^F_{\mathscr{C},\mathscr{S}}} a$  if and only if  $a \in F$ . We use the entailment  $\models_{\text{SEM}_{\mathscr{C},\mathscr{S}_0}}$  and  $=$ SEM<sub>CS1</sub> for confluent rewriting system *CS*<sub>0</sub> and *CS*<sub>1</sub> respectively; and the form  $\models$ SEM<sub>CS</sub> to indicate that any rewriting system can be used.

#### 3 Semantic and NAF-arguments

Let us introduce a formal definition of semantic arguments.

<span id="page-1-0"></span>**Definition 1 (Semantic argument)** *Given a normal logic program P and S*  $\subseteq$  *P. Arg<sub>P</sub>* =  $\langle S, g \rangle$  *is a semantic argument under SEM<sub>C.S</sub> w.r.t. P, if the following conditions hold true:* 

*1.*  $S \models_{\mathit{SEM}^T_{\mathscr{C}\mathscr{S}}} \mathscr{S}$ 

*2. S is minimal w.r.t. the set inclusion satisfying 1.*

We simplify the notation of these semantic arguments as  $\mathscr{A}rg^+$ . Condition 1 states that the interpretation of conclusion *g* is *true w.r.t.* T in SEM<sub>CP</sub> $(S)$ . Condition 2 in Definition [1](#page-1-0) guarantees the support minimality.

Now, let us define NAF-arguments as follows:

<span id="page-1-1"></span>**Definition 2 (NAF-arguments)** *Given a normal logic program P and*  $S \subseteq P$ *. ArgP* =  $\langle S, not g \rangle$  *is a NAF-argument under the SEM* $g \propto w$ .r.t. P, if the following conditions hold true:

- *1.*  $S \models_{\mathit{SEM}^F_{\mathscr{C}\mathscr{S}}} \mathscr{S}$
- *2. S is minimal w.r.t. the set inclusion satisfying 1.*

Condition 1 in Definition [2](#page-1-1) is the interpretation of the conclusion *w.r.t.*  $\models_{\text{SEM}^F_{\mathscr{C},\mathscr{S}}}$ , with the set of all the NAF-arguments noted as  $\mathscr{A}rg^-$ . The addition of *not* in the conclusion of a NAF-argument stresses that such an atom is interpreted as false by  $SEM_{\mathscr{C},\mathscr{S}}$ .

<span id="page-1-3"></span>**Example 1** Let us consider a program  $P_3$  for building semantic and NAF-arguments considering  $\mathcal{CP}_0$ *and*  $\mathscr{CF}_1$ *.* 

*We build semantic and NAF-arguments as follows: 1) get related clauses of atoms (Si); 2) for every related clause compute*  $SEM_{\mathscr{C}\mathscr{S}_0}(S_i)$  *and*  $SEM_{\mathscr{C}\mathscr{S}_1}(S_i)$ *; 3) the support (every*  $S_i$ *) is joined to the conclu* $s$ ion<sup>[1](#page-1-2)</sup>. Then, the following sets of arguments are built considering  $\mathscr{CF}_{0}$  and  $\mathscr{CF}_{1}$ :

<span id="page-1-2"></span><sup>&</sup>lt;sup>1</sup>We implemented this procedure and the sources are open, then can be found in  $\text{https://people.cs.umu.se/~seteban/}$ [argumentation/](https://people.cs.umu.se/~esteban/argumentation/)

$$
P_3 := \begin{cases} a \leftarrow b, not \ c \\ b \leftarrow \\ d \leftarrow not \ d \\ e \leftarrow e \end{cases} \qquad \qquad \underbrace{\begin{pmatrix} A_2^+ = \langle \{b \leftarrow \}, b \rangle \right) & \text{Sematic Attack} \\ \text{d} \leftarrow not \ d \\ \text{d} \bar{1} = \langle \{a \leftarrow b, not \ c \}, not \ b \rangle \end{pmatrix} \end{cases}
$$

Case 
$$
\mathcal{CP}_0
$$
:  $\mathcal{A}_{r}g_{P_3} = \{A_1^+, A_2^+, A_3^+, A_1^-, A_2^-, A_3^-, A_4^-, A_6^-\}$ .  
Case  $\mathcal{CP}_1$ :  $\mathcal{A}_{r}g_{P_3} = \{A_1^+, A_2^+, A_3^+, A_5^+, A_1^-, A_2^-, A_3^-, A_4^-, A_6^-\}$ .

An effect of interpreting argument supports under SEM $_{\mathscr{C}}\mathscr{I}$  is that some atoms (or sets of them) are evaluated in *opposition* to other arguments (*e.g.*,  $A_1^+ = \langle S_2, a \rangle$  and  $A_1^- = \langle S_1, not a \rangle$  in Example [1\)](#page-1-3), suggesting a *semantic attack* relationship.

**Definition 3 (Semantic attack)** *Let*  $A = \langle S_A, a \rangle \in \mathcal{A}$  *rg*<sup>+</sup>,  $B = \langle S_B, not \ b \rangle \in \mathcal{A}$  *rg*<sup>−</sup> *be two semantic ar*guments where  $SEM_{\mathscr{C}}(S_A) = \langle T_A, F_A \rangle$  and  $SEM_{\mathscr{C}}(S_B) = \langle T_B, F_B \rangle$ . We say that A attacks B if  $x \in T_A$  and  $x \in F_B$ *, denoted* attacks(*x*, *y*).

Lemma 1 *Semantic and NAF-arguments built from any normal logic program are always consistent.*

<span id="page-2-1"></span>Definition 4 (Semantic Argumentation Framework (SAF)) *Let P be a normal program. Then, a semantic argumentation framework is the tuple:*  $SAF_P = \langle \mathcal{A} \, r g_P, \mathcal{A} \, t t \rangle$ 

We can straightforward extend the definitions of *argumentation semantics* in [\[7\]](#page-3-7) as follows:

<span id="page-2-2"></span>**Definition 5** Let SAF<sub>P</sub> =  $\langle \mathcal{A} r g_P, \mathcal{A} t t \rangle$  be a semantic argumentation framework. An admissible set of *arguments S* ⊆ *AR is:*

- stable *if and only if S attacks each argument which does not belong to S.*
- preferred *if and only if S is a maximal (*w.r.t. *inclusion) admissible set of AF.*
- complete *if and only if each argument, which is acceptable with respect to S, belongs to S.*
- grounded *if and only if S is the minimal (*w.r.t. *inclusion)* complete *extension of AF*[2](#page-2-0) *.*

**Example 2** Let us consider  $P_5 = \{a \leftarrow not \ b; \ b \leftarrow not \ a; \ c \leftarrow not \ c, not \ a; \ d \leftarrow not \ d, not \ b; \}$ . SEMegg *will remove rules involving atoms c and d. Then, applying Definition [4,](#page-2-1) we have the framework:*  $SAF_{P_5}$  *=*  $\langle \{A_6^- = \langle \{a \leftarrow not \ b\}, \ not \ a \rangle \},A_6^+ = \langle \{b \leftarrow not \ a\}, \ b \rangle, \ A_5^- = \langle \{b \leftarrow not \ a\}, \ not \ b \rangle, \ A_5^+ = \langle \{a \leftarrow not \ a\}, \ b \rangle, \ A_5^+ = \langle \{a \leftarrow not \ a\}, \ b \rangle, \ A_5^+ = \langle \{a \leftarrow not \ a\}, \ b \rangle, \ A_5^+ = \langle \{a \leftarrow not \ a\}, \ b \rangle, \ A_5^+ = \langle \{a \leftarrow not \ a\}, \ b \rangle, \ A_5^+ = \langle \{a \leftarrow$ *not*  $b$ ,  $a$ ,  $\}$ , attacks $(A_5^+)$  $, A_6^+$  $_6^+), \ a t \, t \, a c k s (A^+_5)$  $+$ <sub>5</sub><sup> $+$ </sup>,  $A_6^ \frac{1}{6}$ ), attacks( $A_6^+$  $A_5^+, A_5^+$  $_5^+),$  attacks $(A_6^+$  $+$ <sub>6</sub><sup>+</sup>, $A_5^-$ 5 )⟩*. When we apply Definition [5](#page-2-2) to SAFP*<sup>5</sup> *we obtained the following extensions:*

- *Stable = preferred:*  $\{A_5^+\}$  $+$ <sub>5</sub><sup> $+$ </sup>,  $A_5^ \{A_6^+\}$ ,  $\{A_6^+\}$  $\frac{+}{6}$ ,  $A_6^ \begin{matrix} - \\ 6 \end{matrix} \}$
- *Complete:*  $\{A_5^+\}$  $+$ <sub>5</sub><sup> $+$ </sup>,  $A_5^ \{A_6^+\}$ ,  $\{A_6^+\}$  $+$ <sub>6</sub><sup> $+$ </sup>,  $A_6^ \begin{bmatrix} - \\ 6 \end{bmatrix}, \{\}\}$
- *Grounded:* {}

### 4 Conclusions

The main contributions are: 1) *Semantic Argumentation Frameworks* (SAF) can be used for justifying *true* and *false* interpreted conclusions. 2) SAF is based on *families* of rewriting confluent systems. 3) Satisfies all the well-known argumentation postulates [\[1,](#page-3-8) [4\]](#page-3-9). Future work will involve the exploration of our framework under other Confluent Logic Programming Systems, the satisfaction of other argumentation principles, and the investigation of commonalities between ABA and semantic argumentation.

<span id="page-2-0"></span> ${}^{2}$ In [\[2\]](#page-3-10) it is shown that grounded can be characterized in terms of complete extensions.

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