

Approximating agreements in formal argumentation dialogues¹

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Abstract. In many real applications, to reach an agreement between the participants of a dialogue, which can be for instance a negotiation, is not easy. Indeed, there are application domains such as the medical domain where to reach a consensus among medical professionals is not feasible and might be even regarded as counterproductive. In this paper, we introduce an approach for expressing qualitative preferences between the goals of a dialogue considering ordered disjunction rules. By applying argumentation semantics and degrees of satisfaction of goals, we introduce the so-called *dialogue agreement degree*. Moreover, by considering sets of dialogue agreement degrees, we define a *lattice of agreement degrees*. We argue that a lattice of agreement degrees suggests different approximations between the current state of a dialogue and its aimed goals; hence, a lattice of agreement degrees can help to define different heuristics in the settings of strategic argumentation.

Keywords: Formal dialogues, Non-monotonic reasoning, Logic programming, Goal reasoning, Strategic argumentation

1. Introduction

Formal argumentation has been revealed as a powerful conceptual tool for exploring the theoretical foundations of reasoning and interaction in autonomous systems and multiagent systems [1, 33]. Different dialogue frameworks have been proposed by considering formal argumentation. Indeed, by considering formal argumentation, the so-called *Agreement Technologies* have been introduced in order to deal with the new requirement of interaction between autonomous systems and multiagent systems [28].

Formal argumentation dialogues have been intensively explored in the last years [5, 12, 20, 29, 31] by the community of formal argumentation theory. Most current approaches have been suggested as general frameworks for setting up different kinds of dialogues. Roughly speaking, we can understand a dialogue as a finite sequence of utterances: $[u_1, \dots, u_n]$. Depending on the dialogue approach [5, 12, 20, 29, 31], the sequence of utterances follows a protocol of valid moves performed by the participants of a dialogue. Moreover, these approaches are mainly oriented to a particular topic/goal that is usually denoted by a logical formula. Hence, these dialogue approaches are only concerned about validating a particular goal, *i.e.* a given

logical formula. Therefore, we can say that these approaches were defined for validating only static goals. This means that there is an agreement at the end of a dialogue upon whether the given goal holds true in the outcomes of the dialogue; otherwise, there is no agreement at the end of the dialogue.

In many real applications, to reach an agreement between the participants of a dialogue is not easy [34, 35]. Indeed, there are application domains such as the medical domain where to reach a consensus among medical professionals is not feasible and might even be regarded as counterproductive [19]. In order to illustrate this situation, let us consider a hypothetical scenario from a medical domain in the field of human organ transplanting (the scenario is reported from [27, 35]):

Scenario 1

*Let us assume that we have two transplant coordinators, one of them is against the viability of the organ (TCA_D) and the other is in favour of the viability of the organ (TCA_R). TCA_D argues that the organ is not viable since the donor had endocarditis due to *Streptococcus viridans*, then the recipient could be infected by the same microorganism. In contrast, TCA_R argues that the organ is viable because the organ presents correct function and correct structure and the infection could be prevented with post-treatment with penicillin,*

¹This paper is a revised version of the paper [24].

1 even if the recipient is allergic to penicillin, there is the
2 option of post-treatment with teicoplanin.

3 In the settings of the aforementioned scenario, one
4 can argue that the main goal is to keep alive the re-
5 cipient; however, finding safe-organs is an issue for a
6 discussion between doctors since there are not unique
7 criteria for selecting safe-organs [35].

8 We argue that managing *dynamic degrees of agree-*
9 *ment* during a dialogue can help with the manage-
10 ment of disagreements during a dialogue. These dy-
11 namic degrees of agreement can be defined by consid-
12 ering preferences between the goals of a dialogue. Cur-
13 rently, dialogue systems manage mainly static goals
14 that are usually introduced as the topic of a dialogue
15 [5, 12, 20, 29, 31]. Hence, these approaches do not al-
16 low the specification of preferences between the goals
17 of a given dialogue.

18 Depending on the application domain, we can argue
19 that there are *static* and *dynamic goals* during a dia-
20 logue. A static goal is a goal that cannot be skipped
21 during a dialogue and a dynamic goal is a goal that
22 can change during a dialogue, *e.g.*, a goal that can be
23 skipped during a dialogue. These assumptions suggest
24 a need for defining methods that can manage *degrees*
25 *of agreement* on an ongoing dialogue *w.r.t.* each in-
26 tended goal of a dialogue. In these settings, some re-
27 search questions arise:

28
29 **Q1:** Given a dialogue, is there a *partial degree of*
30 *agreement* between the participants of a dialogue?

31 **Q2:** Given a dialogue, can we *dismiss goals* in order
32 to maximize agreements *w.r.t.* other goals?

33
34 In this paper, we address the aforementioned ques-
35 tions. To this end, we follow Dung style [10] for se-
36 lecting arguments from a set of arguments with dis-
37 agreements. We consider structured arguments, which
38 are constructed from extended logic programs.

39 In order to express preferences between goals that
40 are context-dependent, we consider a qualitative ap-
41 proach for expressing preference namely *logic pro-*
42 *grams with ordered disjunctions*[8]. Hence, logic pro-
43 grams with ordered disjunctions are considered for ex-
44 pressing preferences between the goals of a dialogue.
45 For instance, a possible representation of the dialogue
46 of Scenario 1 is:

$$47 \quad D = \langle Participants, Goals, Utterances \rangle$$

48
49 in which $Participants = \{TCA_D, TCA_R\}$ and

$$50 \quad Goals = \{keep_alive_recipient \leftarrow \top; \\ 51 \quad \quad \quad healthy_donor \leftarrow \top; \\ \quad \quad \quad safe_organs \times managed_disease \leftarrow \top\}.$$

Let us observe that the rule

$$safe_organs \times managed_disease \leftarrow \top$$

suggests that the dialogue looks for safe organs to be
transplanted; however, if not possible, the doctors will
prefer organs that can be treated post-transplanting.
 $Utterances = [u_1, \dots, u_n]$ in which each $u_i (1 \leq i \leq n)$
is an utterance from either TCA_D or TCA_R .

By considering dialogues, argumentation semantics
and subsets of goals, we introduce the so-called *dia-*
logue agreement degree. A dialogue agreement degree
considers different sets of goals such that each goal has
a satisfaction agreement degree in terms of satisfac-
tion degrees of ordered-disjunction rules. Considering
sets of dialogue agreement degrees, we define a *lat-*
tice of agreement degrees. We consider that both dia-
logue agreement degrees and lattices of agreement de-
grees are novel ideas that have not been explored in the
settings of formal argumentation dialogue before. In-
deed, to the best of our knowledge, we are introduc-
ing the first argumentation dialogue system that con-
siders degrees of agreements based on qualitative pref-
erences among the goals of a dialogue. We argue that a
lattice of agreement degrees suggests different approx-
imations between the current state of a dialogue and
its aimed goals. Indeed, a lattice of agreement degrees
can show evidence about whether or not it is accept-
able to dismiss goals in order to maximize agreements
regarding other goals.

The rest of the paper is organized as follows: In sec-
tion 2, basic concepts of logic programming and an
approach for building arguments from logic programs
are presented. In Section 3, our approach for defining
dialogues considering preferences between the goals
of a dialogue is introduced. In Section 4, the concepts
of dialogue agreement degree and lattice of agreement
degrees are formalized. In Section 5, the strategic ar-
gumentation problem is characterized in terms of ex-
tended logic programs and the well-founded seman-
tics. In the last section, our conclusions and future
work are outlined.

2. Background

In this section, a basic background in logic program-
ming is presented. Mainly, extended logic programs

and logic programs with ordered disjunctions are presented. We are assuming that the reader is familiar with basic concepts of Answer Set Programming (ASP). A good introduction to ASP is presented in [2]. In terms of argumentation, we present an approach for building arguments from an extended logic program.

2.1. Extended logic programs

Let us introduce the language of a propositional logic, which consists of propositional symbols: p_0, p_1, \dots ; connectives: $\leftarrow, \neg, \text{not}, \top$; and auxiliary symbols: $(,)$, in which \wedge, \leftarrow are 2-place connectives, \neg , not are 1-place connectives and \top is a 0-place connective. The propositional symbols, the 0-place connective \top and the propositional symbols of the form $\neg p_i$ ($i \geq 0$) stand for the indecomposable propositions, which we call *atoms*, or *atomic propositions*. The atoms of the form $\neg a$ are also called *extended atoms* in the literature. In order to simplify the presentation, we call them atoms as well. The negation symbol \neg is regarded as the so-called *strong negation* in the ASP literature [2], and the negation symbol not as *negation as failure*. A literal is an atom, a (called a positive literal), or the negation of an atom $\text{not } a$ (called a negative literal). A (propositional) extended normal clause, C , is denoted:

$$a \leftarrow b_1, \dots, b_j, \text{not } b_{j+1}, \dots, \text{not } b_{j+n} \quad (1)$$

in which $j + n \geq 0$, a is an atom, and each b_i ($1 \leq i \leq j + n$) is an atom. We use the term *rule* as a synonym of *clause* indistinctly. When $j + n = 0$, the clause is an abbreviation of $a \leftarrow \top$ (a *fact*), such that \top is the propositional atom that always evaluates to true. In a slight abuse of notation, we sometimes write a clause $C = a \leftarrow \mathcal{B}^+ \wedge \text{not } \mathcal{B}^-$, in which $\mathcal{B}^+ := \{b_1, \dots, b_j\}$ and $\mathcal{B}^- := \{b_{j+1}, \dots, b_{j+n}\}$. We denote by $\text{head}(C)$ the head atom a of clause C .

An extended logic program P is a finite set of extended normal clauses. When $n = 0$, the clause is called an *extended definite clause*. By \mathcal{L}_P , we denote the set of atoms that appear in P .

Let A be a set of atoms and P be an extended (definite or normal) logic program. $C = a_0 \leftarrow \mathcal{B}^+, \text{not } \mathcal{B}^- \in P$ is applicable in A if $\mathcal{B}^+ \subseteq A$. $\text{App}(A, P)$ denotes the subset of rules of P which are applicable in A . $C = a_0 \leftarrow \mathcal{B}^+, \text{not } \mathcal{B}^- \in P$ is closed in A if C is applicable in A and $\text{head}(C) \in A$.

Since we are using a comma for denoting the \wedge binary connective in the body of the rules, we will use semicolon for separating elements in sets of rules.

2.2. Logic Programs with Ordered Disjunction

The formalism of *Logic Programs with Ordered Disjunction* (LPODs) was created with the idea of expressing explicit context-dependent preference rules, which select the most plausible atoms to be used in a reasoning process and to order answer sets [8].

Technically speaking, LPODs are based on extended logic programs augmented by an ordered disjunction connector \times which allows for the expression of qualitative preferences in the head of rules [8]. An LPOD is a finite collection of rules of the form:

$$r = c_1 \times \dots \times c_k \leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_{m+n} \quad (2)$$

where c_i 's ($1 \leq i \leq k$) and b_j 's ($1 \leq j \leq m + n$) are atoms. The intuitive reading behind a rule like (2) is that if the body of r is satisfied, then some c_i must be true in an answer set, if possible c_1 , if c_1 is not possible then c_2 , and so on. As previously stated, from a nonmonotonic reasoning point of view, each of the c_i 's can represent alternative ranked options for selecting the most plausible (default) rules of an LPOD.

The LPODs semantics was defined in terms of split programs. Split programs are a way to represent every option of ordered disjunction rules with the property that the set of all answer sets of an LPOD corresponds exactly to the answer sets of the split programs. An alternative and more straightforward characterization of the LPODs semantics was also given in terms of a program reduction defined as follows:

Definition 1 (\times -reduction). [8] *Let $r = c_1 \times \dots \times c_k \leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_{m+n}$ be an ordered disjunction rule and M be a set of atoms. The \times -reduction of a rule r is defined as:*

$r_{\times}^M = \{c_i \leftarrow b_1, \dots, b_m \mid c_i \in M \text{ and } M \cap (\{c_1, \dots, c_{i-1}\} \cup \{b_{m+1}, \dots, b_{m+n}\}) = \emptyset\}$
The \times -reduction is generalized to an LPOD P in the following way:

$$P_{\times}^M = \bigcup_{r \in P} r_{\times}^M$$

Based on the \times -reduction, the LPODs semantics is defined as follows:

Definition 2 (SEM_{LPOD}). [8] *Let P be an LPOD and M be a set of atoms. Then, M is an answer set of P if and only if M is closed under all the rules in P and M is the minimal model of P_{\times}^M . We denote by $SEM_{LPOD}(P)$ the set of answer sets of P .*

One interesting characteristic of LPODs is that they provide a mean to represent preferences among answer sets by considering the satisfaction degree of an answer set w.r.t. a rule [8].

Definition 3 (Rule Satisfaction Degree). [8] *Let M be an answer set of an LPOD P . The satisfaction degree M w.r.t. a rule $r = c_1 \times \dots \times c_k \leftarrow b_1, \dots, b_m$, not $b_{m+1} \dots$, not b_{m+n} , denoted by $deg_M(r)$, is*

- 1 if $b_j \notin M$ for some j ($1 \leq j \leq m$), or $b_i \in M$ for some i ($m+1 \leq i \leq m+n$),
- j ($1 \leq j \leq k$) if all $b_l \in M$ ($1 \leq l \leq m$), $b_i \notin M$ ($m+1 \leq i \leq m+n$), and $j = \min\{r \mid c_r \in M, 1 \leq r \leq k\}$.

The degrees can be viewed as penalties, as a higher degree expresses a lesser degree of satisfaction. Therefore, if the body of a rule is not satisfied, then there is no reason to be dissatisfied and the best possible degree 1 is obtained [8]. A preference order on the answer sets of an LPOD can be obtained by means of the following preference relation.

Definition 4. [8] *Let P be an LPOD, and M_1 and M_2 be two answers of P . M_1 is preferred to M_2 (denoted by $M_1 >_P M_2$) if and only if $\exists r \in P$ such that $deg_{M_1}(r) < deg_{M_2}(r)$ and $\nexists r' \in P$ such that $deg_{M_2}(r') < deg_{M_1}(r')$.*

2.3. Constructing arguments from extended logic programs

In this section, an approach for building arguments from a logic program is presented [17]. In the construction of these arguments, the well-founded semantics (WFS) is used [14]. The well-founded semantics is considered as an approximation of the stable model semantics [15]; moreover, it has the nice property of being polynomial time computable for function-free logic programs.

A definition of the well-founded semantics is presented in Appendix A. Let us observe that WFS is a three-valued semantics that infers a unique partial interpretation of a given logic program. Hence, given a logic program P , $WFS(P) = \langle T, F \rangle$ such that the atoms that appear in T are considered true, the atoms that appear in F are considered false, and the atoms that are neither in T nor in F are considered undefined.

The following definition introduces an approach for constructing arguments from an extended normal logic program.

Definition 5. [17] *Given an extended logic program P and $S \subseteq P$, $Arg_P = \langle S, g \rangle$ is an **argument**, if the following conditions hold:*

- (1) $WFS(S) = \langle T, F \rangle$ such that $g \in T$.
- (2) S is minimal w.r.t. the set inclusion satisfying 1.
- (3) $\nexists g \in \mathcal{L}_P$ such that $\{g, \neg g\} \subseteq T$ and $WFS(S) = \langle T, F \rangle$.

By $Arg(P)$ we denote the set of all of the arguments built from P .

Given an argument $A = \langle S, g \rangle$, S is usually called the *support* of A , g the *conclusion* of A . For the sake of simplicity of some definition, the following projections are defined $CI(A) = g$, and $Sp(A) = S$.

Given a set of arguments Ag , Δ_{Ag} denotes the set of conclusions of the arguments of Ag , i.e. $\Delta_{Ag} = \{CI(A) \mid A \in Ag\}$.

Let us mention that there are other approaches for constructing arguments from a logic program [6, 10, 13, 22, 25, 32]. We are considering an approach that has shown to be a conservative approach since it does not allow problematic arguments such as the self-attacked arguments. For instance, Definition 5 will not construct arguments such as the argument $arg_1 = \langle \{a \leftarrow \text{not } a\}, a \rangle$; nevertheless, arg_1 can be constructed by other approaches for constructing arguments [32]. In the argumentation literature, arg_1 is understood as a self-attacked argument.

Formally, attacks between arguments are binary relations between arguments; moreover, these binary relations express disagreements between arguments. Intuitively, an attack between two arguments emerges whenever there is a *disagreement* between these arguments. Attacks between arguments can be identified by the following definition:

Definition 6 (Attack relationship between arguments). [17] *Let $A = \langle S_A, g_A \rangle$, $B = \langle S_B, g_B \rangle$ be two arguments such that $WFS(S_A) = \langle T_A, F_A \rangle$ and $WFS(S_B) = \langle T_B, F_B \rangle$. We say that A attacks B , denoted by (A, B) , if one of the following conditions holds:*

- (1) $a \in T_A$ and $\neg a \in T_B$.
- (2) $a \in T_A$ and $a \in F_B$.

$At(Arg)$ denotes the set of attack relationships between the arguments belonging to the set of arguments Arg .

1 It has been shown that this definition of attack be-
 2 tween arguments generalizes other definitions of at-
 3 tacks between arguments based on logic programs
 4 [23].

5 Like Dung's style [10], we define the resulting argu-
 6 mentation framework from a logic program as follows:
 7

8 **Definition 7.** Let P be an extended logic program. The
 9 resulting argumentation framework w.r.t. P is the tuple:
 10 $AF_P = \langle Arg_P, At(Arg_P) \rangle$.
 11

12 Following Dung's style [10], argumentation seman-
 13 tics are used for selecting arguments from argumenta-
 14 tion frameworks that were constructed from logic pro-
 15 grams. An argumentation semantics σ is a function that
 16 assigns to an argumentation framework AF_P w.r.t. P a
 17 set of sets of arguments denoted by $\mathcal{E}_\sigma(AF_P)$. Each set
 18 of $\mathcal{E}_\sigma(AF)$ is called σ -extension. Let us observe that σ
 19 can be instantiated with any of the argumentation seman-
 20 tics that has been defined in terms of abstract argu-
 21 ments [3].
 22
 23
 24

25 3. Dialogues and relations between them

26 In this section, we introduce an approach for defin-
 27 ing dialogues between agents. This class of dialogues
 28 will have the property of capturing preferences be-
 29 tween the goals of the dialogues by using ordered dis-
 30 junction programs. As was argued in Section 1, the
 31 main aim of this paper is to study the outcomes (*i.e.*
 32 *agreements*) of an ongoing dialogue by considering the
 33 current *active knowledge*² of a dialogue and the set of
 34 goals of this dialogue. Hence, we put less attention to
 35 the protocols that lead the moves of the participants of
 36 a dialogue. The protocols that lead the moves of the
 37 participants of a dialogue mainly depend on the kind
 38 of dialogue that a dialogue-based system aims to im-
 39 plement [29, 30].
 40

41 Let us start by introducing the basic piece of a dia-
 42 logue that is called *utterance*.
 43

44 **Definition 8.** An utterance of a given agent a is a tuple
 45 of the form $\langle a, A \rangle$ in which A is an argument according
 46 to Definition 5.
 47
 48

49 ²By active knowledge, we mean the information that has been
 50 shared by the participants of a dialogue. Hence, it is assumed that all
 51 the participants of a dialogue have access to this shared information.

1 For the sake of simplicity of presentation, the fol-
 2 lowing notation is introduced. Given an utterance $u =$
 3 $\langle a, A \rangle$, $u^* = A$. Given a set of utterances \mathcal{U} , $\mathcal{U}^* =$
 4 $\{u^* \mid u \in \mathcal{U}\}$.

5 An utterance is a suggested argument by an agent
 6 a in an ongoing dialogue. Considering utterances, dia-
 7 logues between a set of agents are defined as follows:
 8

9 **Definition 9.** A dialogue is a tuple of the form $\langle \mathcal{I}, G, D_r^t \rangle$
 10 in which G is a logic program with ordered disjunction
 11 and D_r^t is a finite sequence of utterances $[u_r, \dots, u_t]$ in-
 12 volving a set of participating agents \mathcal{I} , where $r, t \in \mathbb{N}$
 13 and $r \leq t$, such that:
 14

- 15 (1) $Sender(u_s) \in \mathcal{I}$ ($r \leq s \leq t$),

16 in which $Sender : \mathcal{U} \mapsto \mathcal{I}$ is a function such that
 17 $Sender(u) = Agent$, $u \in \mathcal{U}$ and \mathcal{U} denotes the set of
 18 all the possible utterances of the participating agents
 19 \mathcal{I} .
 20

21 In order to project the utterances shared in a dia-
 22 logue, let us introduce the following notation: given a
 23 dialogue, $D = \langle \mathcal{I}, G, [u_r, \dots, u_t] \rangle$, $\mathcal{U}_D = \{u_i \mid r \leq i \leq$
 24 $t, [u_r, \dots, u_t]\}$.

25 Definition 9 is illustrated by following simple ab-
 26 stract example.
 27

28 **Example 1.** Let $D_1 = \langle \mathcal{I}, G, D_1^2 \rangle$ such that $\mathcal{I} =$
 29 $\{1, 2\}$, $G = \{a \times c \leftarrow \top; b \leftarrow \top\}$, $D_1^2 = [u_1, u_2]$,
 30 $u_1 = \langle 1, \langle \{a \leftarrow not\ b\}, a \rangle \rangle$ and $u_2 = \langle 2, \langle \{c \leftarrow$
 31 $\top; b \leftarrow c\}, b \rangle \rangle$. Hence, D_1 is a dialogue between two
 32 agents. D_1 has as goals the topics expressed in terms
 33 of two ordered disjunction rules: $a \times c \leftarrow \top$ and
 34 $b \leftarrow \top$. D_1 has two utterances: u_1, u_2 . We can see that
 35 $\mathcal{U}_{D_1} = \{u_1, u_2\}$.
 36

37 Let us observe that given a dialogue D , we can get an
 38 *active knowledge base*, *i.e.* an extended logic program,
 39 w.r.t. D . Moreover, we can get the set of conclusions of
 40 the utterances w.r.t. D .
 41

42 **Definition 10.** Let $D = \langle \mathcal{I}, G, U_r^t \rangle$ be a dialogue.

- 43
- 44 • The active knowledge base w.r.t. D , denoted by
 - 45 \mathcal{A}_D , is $\mathcal{A}_D = \bigcup_{u \in \mathcal{U}_D} Sp(u^*)$.
 - 46 • The argument-conclusions of the utterances w.r.t.
 - 47 D , denoted by \mathcal{C}_D , is: $\mathcal{C}_D = \bigcup_{u \in \mathcal{U}_D} Cl(u^*)$.
 - 48

49 The active knowledge of a dialogue is the informa-
 50 tion that the participating agents in a dialogue have
 51 shared by means of arguments.

Example 2. Considering the dialogue D_1 introduced by Example 1, we can see that:

$$\begin{aligned} \mathcal{A}_{D_1} &= \{a \leftarrow \text{not } b; c \leftarrow \top; b \leftarrow c\} \\ \mathcal{C}_{D_1} &= \{a, b\} \end{aligned}$$

Considering the information of a dialogue in terms of utterances, active knowledge and arguments, we define four kinds of sub-dialogues.

Definition 11. Let $D = \langle \mathcal{I}, G, U_r^t \rangle$, $D' = \langle \mathcal{I}', G', U_i^j \rangle$ be two dialogues.

- D' is a sub-dialogue w.r.t. utterances of D ($D' \sqsubseteq_u D$) iff $\mathcal{U}_{D'}^* \subseteq \mathcal{U}_D^*$.
- D' is a sub-dialogue w.r.t. active-knowledge of D ($D' \sqsubseteq_{ak} D$) iff $\mathcal{A}_{D'} \subseteq \mathcal{A}_D$.
- D' is a sub-dialogue w.r.t. argument-conclusions of D ($D' \sqsubseteq_{ac} D$) iff $\mathcal{C}_{D'} \subseteq \mathcal{C}_D$.
- D' is a sub-dialogue w.r.t. goals of D ($D' \sqsubseteq_g D$) iff $G' \subseteq G$.

We illustrate Definition 11 in the following example.

Example 3. Let D_1 be the dialogue introduced by Example 1 and $D_2 = \langle \mathcal{I}_2, G_2, D_1^1 \rangle$ such that $\mathcal{I}_2 = \{1, 2\}$, $G_2 = \{a \times c \leftarrow \top; b \leftarrow \top\}$, $D_1^1 = [u_1]$ and $u_1 = \langle 1, \langle \{a \leftarrow \text{not } b\}, a \rangle \rangle$.

We are assuming that D_1 and D_2 have the same participating agents. Following Definition 11, the following sub-dialogue relations hold: $D_2 \sqsubseteq_u D_1$, $D_2 \sqsubseteq_{ak} D_1$, $D_2 \sqsubseteq_{ac} D_1$, $D_2 \sqsubseteq_g D_1$ and $D_1 \sqsubseteq_g D_2$

Given that the definitions of sub-dialogues, introduced by Definition 11, are basically defined in terms of subsets, the equality between dialogues is defined by the classical definition of set-equality.

Definition 12. Let $D = \langle \mathcal{I}, G, U_r^t \rangle$, $D' = \langle \mathcal{I}', G', U_i^j \rangle$ be two dialogues and $\epsilon \in \{u, ak, ac, g\}$. D and D' are ϵ -equal ($D' =_\epsilon D$) iff $D' \sqsubseteq_\epsilon D$ and $D \sqsubseteq_\epsilon D'$ holds.

It is easy to see that if two dialogues are utterances-equal, then they are active-knowledge and argument-conclusions equal.

Proposition 1. Let $D = \langle \mathcal{I}, G, U_r^t \rangle$, $D' = \langle \mathcal{I}', G', U_i^j \rangle$ be two dialogues. If $D' =_u D$, then $D' =_{ak} D$ and $D' =_{ac} D$.

Proof. If $D' =_u D$, then D and D' have the same arguments. Hence, by definition of argument (see Definition 5), the statement holds true. \square

Let us observe that if two dialogues are active-knowledge equal, it does not imply that they are utterances-equal and argument-conclusions-equal. The main reason for this is because two arguments can be constructed with the same conclusion but with different supports. This property is quite common in different approaches for constructing arguments from a knowledge base [6, 21, 32].

Considering a dialogue, two argumentation frameworks can be derived from it.

Definition 13. Let $D = \langle \mathcal{I}, G, U_r^t \rangle$ be a dialogue.

- The resulting argumentation framework AF_D^u w.r.t. D and its utterances is $\langle \mathcal{U}_D^*, At(\mathcal{U}_D^*) \rangle$.
- The resulting argumentation framework AF_D^{ak} w.r.t. D and its active-knowledge is:

$$\langle Arg(\mathcal{A}_D), At(Arg(\mathcal{A}_D)) \rangle$$

AF_D refers to either AF_D^u or AF_D^{ak} .

We can illustrate Definition 13 with the following simple example:

Example 4. Let D_1 be the dialogue introduced by Example 1.

$$AF_{D_1}^u \text{ w.r.t. } D_1 \text{ is } \langle \{arg_1, arg_2\}, \{(arg_2, arg_1)\} \rangle$$

$$AF_{D_1}^{ak} \text{ w.r.t. } D_1 \text{ is } \langle \{arg_1, arg_2, arg_3\}, \{(arg_2, arg_1)\} \rangle$$

in which $arg_1 = \langle \{a \leftarrow \text{not } b\}, a \rangle$, $arg_2 = \langle \{c \leftarrow \top; b \leftarrow c\}, b \rangle$ and $arg_3 = \langle \{c \leftarrow \top\}, c \rangle$.

Let us observe that the arguments of AF_D^u are the arguments that the participating agents of D have explicitly shared by means of utterances in the dialogue. However, by considering the active-knowledge of a dialogue both new arguments and new attacks can emerge; hence, AF_D^{ak} suggests a different view of the shared information in a dialogue. Nevertheless, we can identify a relationship between AF_D^u and AF_D^{ak} .

Proposition 2. Let $D = \langle \mathcal{I}, G, U_r^t \rangle$ be a dialogue, $AF_D^u = \langle A^u, At^u \rangle$ and $AF_D^{ak} = \langle A^{ak}, At^{ak} \rangle$. The following subset relations hold true: $A^u \subseteq A^{ak}$ and $At^u \subseteq At^{ak}$.

Proof. Let us observe that arguments can have sub-arguments. These sub-arguments are explicitly identified by A^{ak} . Hence, it is direct that $A^u \subseteq A^{ak}$ holds true. By having more arguments, we can have new attacks between the arguments of A^u and the new explicit sub-arguments identified by A^{ak} . Therefore, $A^{u'} \subseteq A^{ak}$ holds true. \square

We consider that AF_D^u and AF_D^{ak} show different perspectives of an ongoing dialogue. As we will see in Section 5, there are dialogue game interactions in which the participants of a dialogue has to deal with strategic decisions to decide which information to disclose to achieve its own goals in a dialogue. Hence, AF_D^u and AF_D^{ak} can regarded as explicit and implicit views of an ongoing dialogue that can support strategic decision processes of a rational agent.

4. Agreement degrees of dialogues

Up to now, we have seen how to deal with the information that has been shared by the participating agents in a dialogue in terms of argumentation frameworks; however, we have not seen how this information can be understood regarding the goals of the dialogue.

As was mentioned in the previous section, the shared information in a dialogue can define different argumentation frameworks. Now in this section, we will use these argumentation frameworks for defining the satisfiability of the goals of a given dialogue.

The inference from argumentation frameworks is usually led by considering argumentation semantics. Hence, we will use σ -extensions of a σ argumentation semantics for defining answer sets of ordered disjunction rules as follows:

Definition 14. Let $D = \langle \mathcal{I}, G, U_r^l \rangle$ be a dialogue, $G' \subseteq G$ and σ be an argumentation semantics. A σ -extension $E_\sigma \in \mathcal{E}_\sigma(AF_D)$ is a σ -model of G' iff $M = \mathcal{L}_{G'} \cap \Delta_{E_\sigma}$ is an answer set of G' . $\mathcal{M}_\sigma(AF_D, G')$ denotes the set of all σ -models inferred by the argumentation semantics σ w.r.t. AF_D and G' .

Let us observe, in Definition 14, that the σ argumentation semantics is suggesting sets of atoms that can be considered for satisfying the goals of a dialogue. As was mentioned in Section 2.2, an answer set infers a satisfaction degree of an ordered disjunction rule. Hence, by considering this satisfaction degree of each goal (an ordered disjunction), we define a satisfaction degree of a set of goals as follows:

Definition 15. Let $D = \langle \mathcal{I}, G, U_r^l \rangle$ be a dialogue, $G' \subseteq G$, σ be an argumentation semantics. The satisfaction degree of $M \in \mathcal{M}_\sigma(AF_D, G')$ w.r.t. AF_D and G' is:

$$deg_M(AF_D, G') = \max\{deg_M(r) | r \in G'\}$$

Let us observe that $deg_M(AF_D, G')$ is capturing the satisfaction degree of the ordered disjunction rules that were worst satisfied. It is worth mentioning that according to Definition 4, an ordered disjunction rule with higher degree expresses a lesser degree of preference satisfaction. Hence if a dialogue and an argumentation semantics suggest that the $deg_M(AF_D, G') = 1$, it means that all the goals of G' were satisfied in its best case. However, if $deg_M(AF_D, G') = 2$, it means that at least one of the decisions (*i.e.* an ordered disjunction rule) of G' took the second option.

By considering the satisfaction degree w.r.t. σ models (see Definition 15), we can define preferences between σ models .

Definition 16. Let $D = \langle \mathcal{I}, G, U_r^l \rangle$ be a dialogue, $G' \subseteq G$ and σ be an argumentation semantics. If $M_1, M_2 \in \mathcal{M}_\sigma(AF_D, G')$, M_1 is preferred to M_2 (denoted by $M_1 >_p M_2$) if and only if $deg_{M_1}(AF_D, G') < deg_{M_2}(AF_D, G')$.

One can see that $>_p$ defines a total ordered set by considering all the σ models suggested by an argumentation semantics σ .

Proposition 3. Let $D = \langle \mathcal{I}, G, U_r^l \rangle$ be a dialogue, $G' \subseteq G$ and σ be an argumentation semantics. $(\mathcal{M}_\sigma(AF_D, G'), >_p)$ is a total order set.

Proof. Let us start observing that deg_M is a relation of the form $deg_M : AF_D \times 2^G \mapsto \mathbb{N}$. Hence, if $M_1, M_2 \in \mathcal{M}_\sigma(AF_D, G')$ and $M_1 >_p M_2$, then $\exists n_1, n_2 \in \mathbb{N}$ such that $deg_{M_1}(AF_D, G') = n_1$, $deg_{M_2}(AF_D, G') = n_2$ and $n_1 < n_2$. Then, the proof follows by the fact that \mathbb{N} is a total order set. \square

In [24], we claimed that $(\mathcal{M}_\sigma(AF_D, G'), >_p)$ was only a partial-ordered set, but it is a total order set as it is shown by Proposition 3.

Let us denote by $\max(D, G', \sigma)$ the maximum satisfaction degree of the members of $(\mathcal{M}_\sigma(AF_D, G'), >_p)$.

Now we are ready for defining the dialogue agreement degree suggested by an argumentation semantics σ regarding a given dialogue.

Definition 17 (Dialogue agreement degree). Let $D = \langle \mathcal{I}, G, U_r^i \rangle$ be a dialogue, $G' \subseteq G$ and σ be an argumentation semantics. The dialogue agreement degree of D w.r.t. AF_D and σ (denoted by $D\text{-Deg}(D, AF_D, G', \sigma)$) is a tuple of the form $\langle i/n, \max(D, G', \sigma) \rangle$ such that $i = |G'|$ and $n = |G|$.

According to Definition 17, a dialogue D reaches a total agreement whenever $D\text{-Deg}(D, AF_D, \sigma) = \langle 1, 1 \rangle$, which means that all the goals were satisfied and all of them took the best option.

Example 5. Once again, let us consider the dialogue D_1 introduced by Example 1. Hence, $D_1 = \langle \mathcal{I}, G, D_1^2 \rangle$ such that $\mathcal{I} = \{1, 2\}$, $G = \{a \times c \leftarrow \top; b \leftarrow \top\}$, $D_1^2 = [u_1, u_2]$, $u_1 = \langle 1, \langle \{a \leftarrow \text{not } b\}, a \rangle \rangle$ and $u_2 = \langle 2, \langle \{c \leftarrow \top; b \leftarrow c\}, b \rangle \rangle$.

As we saw in Example 4, $AF_{D_1}^{ak}$ w.r.t. D_1 is $\{\{arg_1, arg_2, arg_3\}, \{\{arg_2, arg_1\}\}\}$ in which $arg_1 = \langle \{a \leftarrow \text{not } b\}, a \rangle$, $arg_2 = \langle \{c \leftarrow \top; b \leftarrow c\}, b \rangle$ and $arg_3 = \langle \{c \leftarrow \top\}, c \rangle$.

If we consider the grounded semantics [10], denoted by gs , $\mathcal{E}_{gs}(AF_{D_1}^{ak}) = \{\{arg_2, arg_3\}\}$. We can see that $\Delta_{\{arg_2, arg_3\}} = \{b, c\}$. Moreover, one can see that $M_{gs} = \mathcal{L}_G \cap \Delta_{\{arg_2, arg_3\}}$ is a gs -model of G .

Let us denote by $r_1 = a \times c \leftarrow \top$ and $r_2 = b \leftarrow \top$. We can see that $deg_{M_{gs}}(r_1) = 2$ and $deg_{M_{gs}}(r_2) = 1$. Therefore, $deg_{M_{gs}}(AF_{D_1}^{ak}, G) = 2$.

Since the grounded semantics only infers a unique gs -model, we get a unique element in $\mathcal{M}_{gm}(AF_{D_1}, G)$. One can see that $D\text{-Deg}(D_1, AF_{D_1}^{ak}, G, gs) = \langle 1, 2 \rangle$. By removing goals from G , one can get different agreement degrees w.r.t. AF_D^{ak} and gs . For instance, by considering the sets $\{a \times c \leftarrow \top\}$ and $\{b \leftarrow \top\}$, we get:

$$\begin{aligned} D\text{-Deg}(D_1, AF_{D_1}^{ak}, \{a \times c \leftarrow \top\}, gs) &= \langle 0.5, 2 \rangle. \\ D\text{-Deg}(D_1, AF_{D_1}^{ak}, \{b \leftarrow \top\}, gs) &= \langle 0.5, 1 \rangle. \end{aligned}$$

In Figure 1, it is depicted the different agreement degrees that can be committed considering the current sequence of utterances of D_1 . Let us point out that Figure 1 suggests different readings regarding dismissing some of the goals of the D_1 . For instance, $D\text{-Deg}(D_1, AF_{D_1}^{ak}, \{b \leftarrow \top\}, gs) = \langle 0.5, 1 \rangle$ suggests that one of the goals is satisfied in its optimal value; however, it is skipping other goals of the dialogue.

One can observe that agreement degree values are monotonic regarding the size of the set of goals.

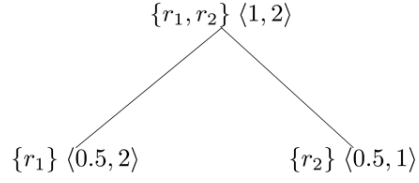


Fig. 1. A lattice of agreement degrees of Example 5.

Proposition 4. Let $D = \langle \mathcal{I}, G, U_r^i \rangle$, $D' = \langle \mathcal{I}', G', U_i^j \rangle$ be two dialogues and σ be an argumentation semantics.

- If $D =_u D'$ and $D' \sqsubseteq_g D$, then $j' \leq j$ such that $D\text{-Deg}(D, AF_D, G, \sigma) = \langle i, j \rangle$ and $D\text{-Deg}(D', AF_{D'}, G', \sigma) = \langle i', j' \rangle$.

Proof. If $D =_u D'$, $\mathcal{M}_\sigma(AF_D, G) = \mathcal{M}_\sigma(AF_{D'}, G')$. Therefore, if a $g \in G$ is satisfied by $M \in \mathcal{M}_\sigma(AF_D, G)$ by degree j , g will be satisfied by a $M' \in \mathcal{M}_\sigma(AF_{D'}, G')$ by degree j . Therefore $deg_M(AF_D, G') \leq j$. \square

As we can see in Figure 1, if we consider all the possible subsets of the set of goals of a dialogue, we can identify different understanding of an ongoing dialogue in terms of agreement degrees. Therefore, by having a list of utterances U_r^i , we can identify the best possible agreements that are possible to reach by considering different subsets of goals. Hence, a lattice of agreement degrees is defined as follows:

Definition 18 (Lattice of agreement degrees). Let $D = \langle \mathcal{I}, G, U_r^i \rangle$ be a dialogue, σ be an argumentation semantics. The lattice of agreement degrees w.r.t. D and σ is $\Omega_D^\sigma = (L, \leq_\Omega)$ in which:

- $L = \{\langle G', \max(D', G', \sigma) \rangle \mid G' \in 2^G \setminus \emptyset, D' = \langle \mathcal{I}, G', U_r^i \rangle\}$
- \leq_Ω is a lexicographical order considering the \subseteq relation for the first element of the tuple and the numerical relation \leq for the last element of the tuple.

Let us observe that one can also define a lattice of agreements by considering all the possible tuples suggested by Definition 17. The unique difference will be the first element of the tuples.

Let us point out that Ω_D^σ is defined in terms of a particular argumentation semantics σ . Nevertheless, by considering different argumentation semantics, one can identify different evaluations of the elements of Ω_D^σ .

Before ending this section, let us mention that the big issue regarding the construction of Ω_D^σ is the computational complexity of the argumentation semantics σ . An important concern in argumentation semantics is their *computational complexity*. It is known that the decision problems of the well-accepted argumentation semantics range from NP-complete to $\Pi_2^{(p)}$ -complete [11].

5. Strategic dialogue argumentation

In this section, the problem of strategic argumentation is presented. The *Strategic argumentation* problem can be regarded as a dialogue game where a player should decide what to move (set of rules) to play in each turn in order to prove (disprove) a given thesis [16]. In these settings, it is assumed that each participant of the dialogue has private knowledge. Whenever one of the participants posts a structured argument $\langle S, a \rangle$ in the dialogue, the support of the argument, which is S , is turned into public information. Hence, the rest of the participants of the dialogue can use S for building their own arguments. This means that any information disclosed by a player can be used against itself.

In the settings of strategic argumentation, the process of deciding which set of rules to disclosure from private knowledge has been shown to be NP-complete even when the problem of deciding whether a given theory entails a literal can be computed in polynomial time [16]. In this section, we extend this result in the settings of strategy argumentation where each participant has its knowledge base captured by extended logic programs.

Let us consider two players: a proponent Pr and an opponent Op . Pr and Op have a private extended logic program, P_{Pr} and P_{Op} respectively. In addition, there is an extended logic program which is public knowledge, denoted by P_{Com} ³. In each move of a dialogue game, Pr and Op add knowledge to P_{Com} by posting arguments at the dialogue. A split extended logic program *w.r.t.* a dialogue game is: $P_s = \langle P_{Com}, P_{Pr}, P_{Op} \rangle$.

The Desired Inference Problem (DIP) *w.r.t.* split extended logic programs

Instance: Let $P_s = \langle P_{Com}, P_{Pr}, P_{Op} \rangle$ be a split program *w.r.t.* a dialogue game and c is a propositional atom.

³Let us observe that P_{Com} is equivalent to \mathcal{A}_D of a given dialogue $D = \langle \mathcal{I}, G, U_r^i \rangle$, see Definition 10.

Question: Let $x \in \{Pr, Op\}$, is there a $S \subseteq P_x$ such that $\langle S', a \rangle$ is an argument according to Definition 5 and $S' \subseteq P_{Com} \cup S$?

Theorem 1. *The Desired Inference Problem w.r.t. split extended logic programs is NP-complete.*

Proof. Let us start introducing the following observation:

- (1) The construction of arguments *w.r.t.* Definition 5 is based on the inference of the well-founded semantics.
- (2) The well-founded semantics is polynomial time computable [14].

The proof follows by Observations 1,2 and Theorem 15 from [16]. \square

The direct implication of Theorem 1 is that each move, of a player in a dialogue game, is computationally expensive in its worst case. This situation is quite critical in real applications. For instance, if the player is taking the role of a *persuasive agent* [18] where the reaction of the persuasive agent has to be done in real-time. Hence, there is a need to define heuristics to optimize the decision process of agents taking part in dialogue games. We consider that structures as a lattice of agreement degrees can help to define heuristics to decide which information to disclose in strategic interactions as the ones in strategic argumentation.

6. Conclusions and future work

Currently, formal argumentation dialogue systems see the disagreements of a dialogue from the perspective of a unique argumentation framework [5, 26]. However, in open environments of agents, the participating agents of a dialogue can join a dialogue and have different interpretations of the shared knowledge by the participating agents. From this perspective, we consider that a given shared knowledge base can give place to different argumentation frameworks. In this article, we show that the active knowledge of a dialogue can give place to at least two different argumentation frameworks AF_D^{ak} , AF_D^u (see Definition 13). Considering Proposition 2, it is easy to see that AF_D^{ak} is an expansion [4] of AF_D^u . We have considered an approach, for constructing arguments, that does not allow us to construct self-attacked arguments. However, considering other constructions of arguments (*e.g.*, [32]),

one can identify different argumentation frameworks from the same active knowledge base of a dialogue. From this perspective, the use of self-attacked arguments can be an interesting topic for defining strategies in order to decide the next moves of an ongoing dialogue.

We show that by considering an argumentation semantics approach we can manage ordered disjunctions rules such that these ordered disjunctions rules capture preferences between goals of a dialogue. We show that argumentation semantics can define different satisfaction degrees of the goals of a dialogue, which are captured by ordered disjunctions rules. Hence, considering the active knowledge of a dialogue and an argumentation semantics, we introduce an approach for measuring an agreement degree of a dialogue. Considering this agreement degree of a dialogue, we introduce an approach for answering the research question **Q1**. Since the agreements of a dialogue are inferred by a given argumentation semantics, one can define different agreement degrees by considering different argumentation semantics. In these settings, the following research question arises:

Q3: Which argumentation semantics infers the maximum (or minimum) agreement degrees of a dialogue and its goals?

Answering **Q3** will be part of our future work. Let us point out that by considering different argumentation semantics one can also define different lattices of agreement degrees. It is known that there are different sub-contention relations between different well-acceptable argumentation semantics [3]. Hence, to see the effect of these sub-contention relations in agreement degrees of dialogues will be also part of our future work.

Considering the lattice of agreement degrees, we introduce an approach for answering **Q2**. Let us observe that $\Omega_D^\sigma = (L, \leq_\Omega)$ shows a picture of the pros and the cons of eliminating goals of a dialogue since L is defining different agreement degrees by considering different subset of goals of the initial set of goals of a dialogue.

Let us point out that in this paper we are introducing a novel approach for modeling dialogues with preferences in their goals. We argue that the satisfaction degrees of a dialogue is a novel approach for defining heuristics to decide the next move in an ongoing dialogue. We have shown that the strategic argumentation problem is also NP-complete in the settings of extended logic programs and the well-founded seman-

tics, see Theorem 1. This result extends the previous results by Governatori *et al.* [16]. We argue that the suggested lattice of agreement degrees can help to define heuristics in the setting of strategic argumentation.

From our applied research, we have observed that considering only static goals in a dialogue does not work in real applications. For instance, let us consider the case of persuasive software agents. If a given persuasive software agent has as a goal to persuade a given human agent, the persuasive software agent will need take into consideration different possible scenarios of agreement where different user preferences can be partially satisfied during a dialogue. Hence, we consider that by modeling preferences between the goals of a dialogue, one can incorporate user preferences into dialogues between software agents and human agents [18].

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Appendix A. Well-Founded Semantics

In order to present a *simple* definition of the Well-Founded Semantics (WFS), we will present a characterization of WFS in terms of rewriting systems. Hence, we define some basic transformation rules for normal logic programs.

Definition 19 (Basic Transformation Rules). [9]

A transformation rule is a binary relation on $Prog_{\mathcal{L}}$. The following transformation rules are called basic. Let a program $P \in Prog_{\mathcal{L}}$ be given.

RED⁺: with This transformation can be applied to P if there is an atom a which does not occur in $HEAD(P)$. **RED⁺** transforms P to the program where all occurrences of *not* a are removed.

RED⁻: This transformation can be applied to P , if there is a rule $a \leftarrow \top \in P$. **RED⁻** transforms P to the program where all clauses that contain *not* a in their bodies are deleted.

Success: Suppose that P includes a fact $a \leftarrow \top$ and a clause $q \leftarrow body$ such that $a \in body$. Then we replace the clause $q \leftarrow body$ by $q \leftarrow body \setminus \{a\}$.

Failure: Suppose that P contains a clause $q \leftarrow body$ such that $a \in body$ and $a \notin HEAD(P)$. Then we erase the given clause.

Loop: We say that P_2 results from P_1 by $Loop_A$ if, by definition, there is a set A of atoms such that:

- (1) for each rule $a \leftarrow body \in P_1$, if $a \in A$, then $body \cap A \neq \emptyset$,
- (2) $P_2 := \{a \leftarrow body \in P_1 \mid body \cap A = \emptyset\}$,
- (3) $P_1 \neq P_2$.

Let CS_0 be the rewriting system such that it contains the transformation rules: RED^+ , RED^- , $Success$, $Failure$, and $Loop$. We denote the uniquely determined

normal form of a program P with respect to the system CS_0 by $norm_{CS_0}(P)$.

WFS was introduced in [14] and was characterized in terms of rewriting systems in [7]. This characterization is defined as follows:

Lemma 1. [7] CS_0 is a confluent rewriting system. It induces a 3-valued semantics that it is the Well-founded Semantics.

Let us observe that Lemma 2.1 is characterizing WFS for the class of normal logic programs. Hence, for using this definition in the class of extended logic programs, we assume that the extended logic programs are transformed into normal logic programs by replacing extended atoms by new symbols.

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