On the hierarchy of block deterministic languages

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History

- 1986: the language SGML is published. Its structure is specified by a Document Type Definition (DTD) using regular expressions where each next position can be decided without look-ahead.
  Examples:
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- 2008: Han, Y.-S., Wood, D.: Generalizations of 1-deterministic regular languages

Overview

1. Preliminaries
   - Basics
   - 1-unambiguous regular languages
   - Block deterministic regular languages
2. Previous results on block deterministic regular languages
   - Determining whether a language L is block deterministic
   - The hierarchy of k-block deterministic regular languages
3. New results
   - Flawed results
   - A witness for the infinite hierarchy

Regular Languages

Regular expression

$$E = \emptyset \qquad E = \varepsilon \qquad E = a$$
$$E = (F + G) \qquad E = (F \cdot G) \qquad E = (F^*)$$

Language denoted by E

$$\mathrm{L}(\emptyset) = \emptyset \qquad \mathrm{L}(\varepsilon) = \{\varepsilon\} \qquad \mathrm{L}(a) = \{a\}$$
$$\mathrm{L}(F + G) = \mathrm{L}(F) \cup \mathrm{L}(G) \qquad \mathrm{L}(F \cdot G) = \mathrm{L}(F) \cdot \mathrm{L}(G) \qquad \mathrm{L}(F^*) = \mathrm{L}(F)^*$$

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Example

- $E = (a + bc)(d + e)$ with $\mathrm{L}(E) = \{ad, ae, bcd, bce\}$
- $F = (a + bc)^*$ with $\mathrm{L}(F) = \{\varepsilon, a, aa, bc, aaa, abc, bca, \ldots\}$

Automaton

Definition

An *automaton* A is a 5-tuple $(\Sigma, Q, I, F, \delta)$ with:

- Σ a finite alphabet
- Q a finite set of states
- $I \subset Q$ the set of initial states
- $F \subset Q$ the set of final states
- $\delta \subset Q \times \Sigma \times Q$ a set of transitions

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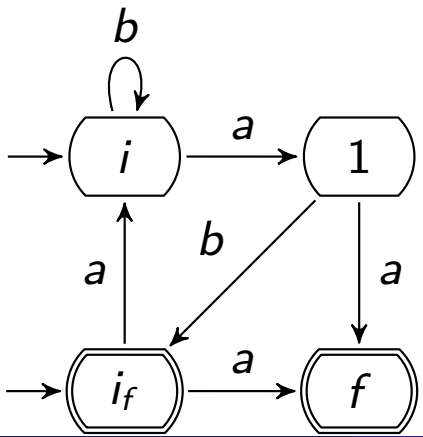
Language recognized

The language recognized by an automaton is the set of labels of all paths that lead from an initial state to a final state.



$$L = \{\varepsilon, a, aa, ab, \ldots\}$$

Deterministic automaton (DFA)

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An automaton $A = (\Sigma, Q, I, F, \delta)$ is *deterministic* if:

- $|I| = 1$
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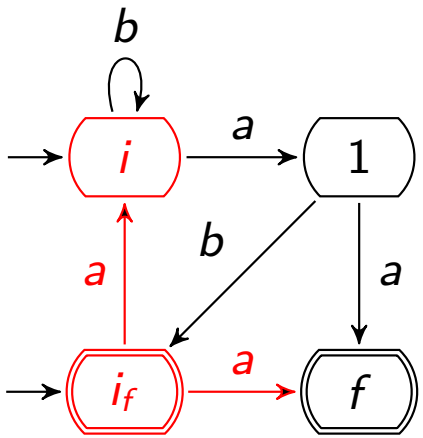

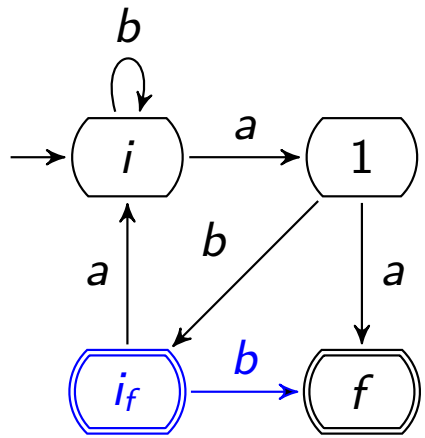

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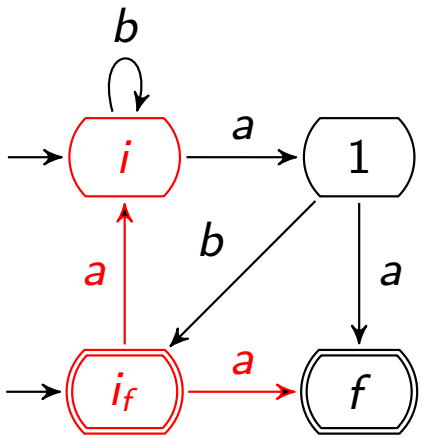
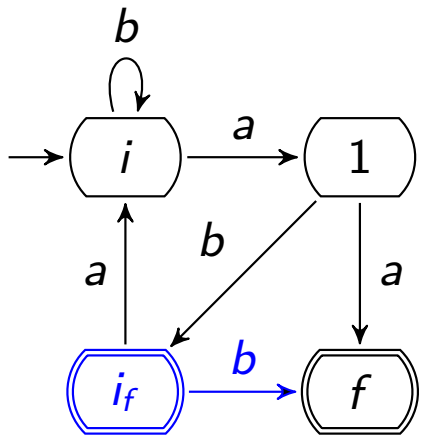




Minimality

A DFA is minimal if there does not exist another DFA recognizing the same language with less states.

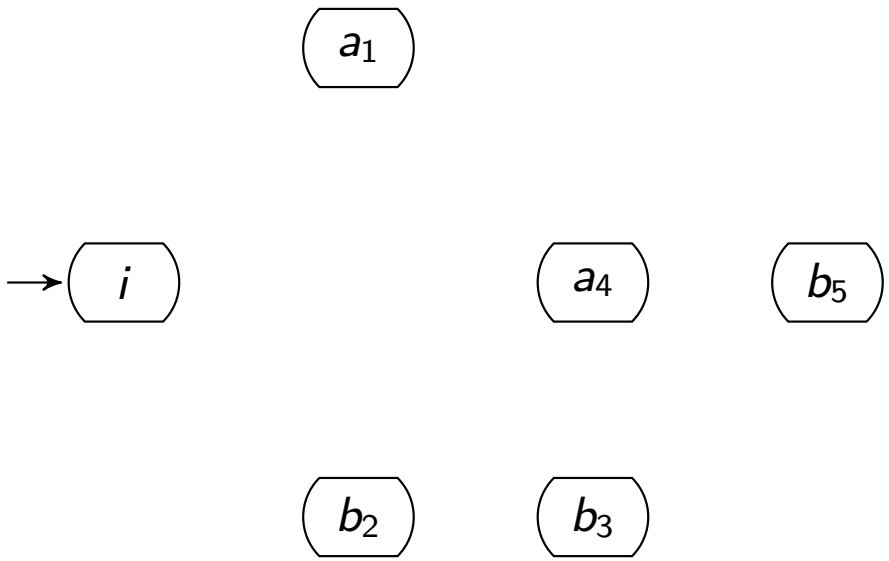
From a regular expression to its Glushkov automaton

$$E = (a + (bb)^*)ab^* + \varepsilon$$

Figure: The Glushkov automaton of E

From a regular expression to its Glushkov automaton

$$E^\sharp = (a_1 + (b_2 b_3)^*) a_4 b_5^* + \varepsilon$$



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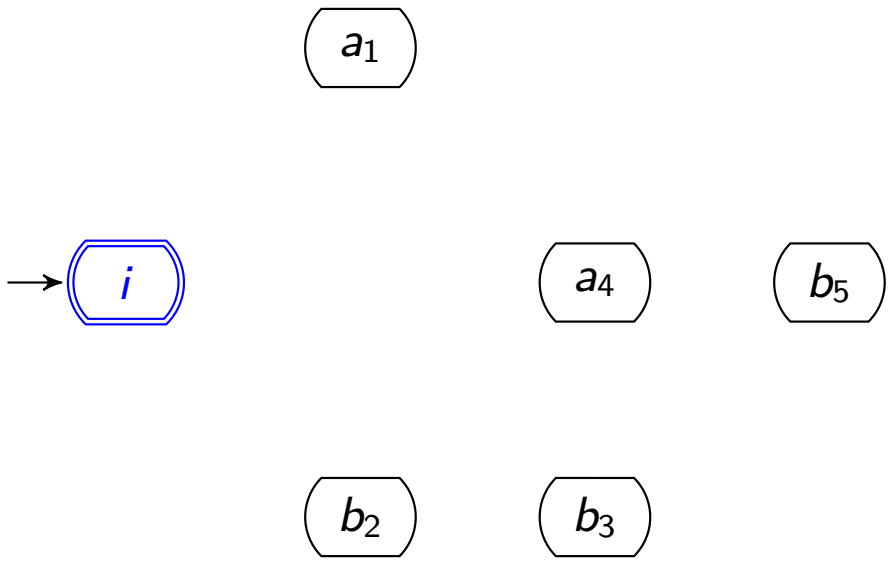

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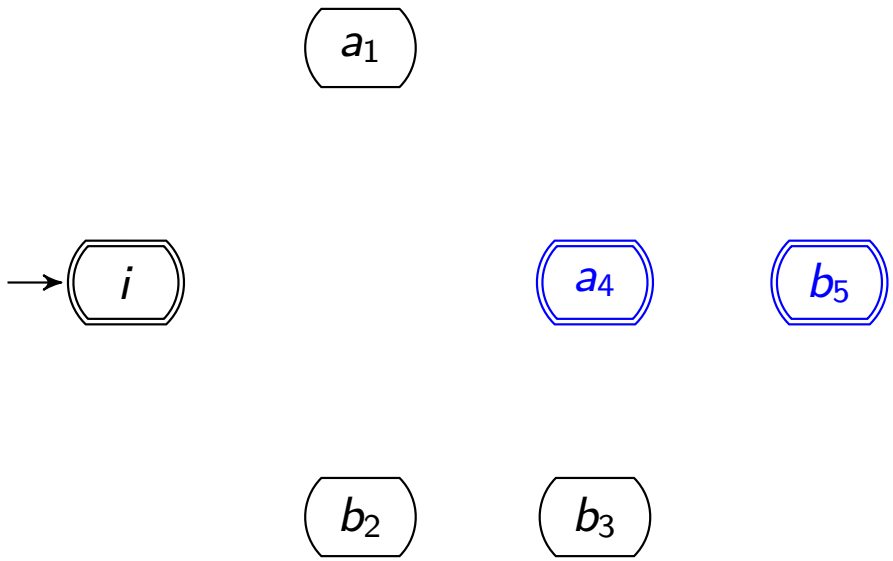

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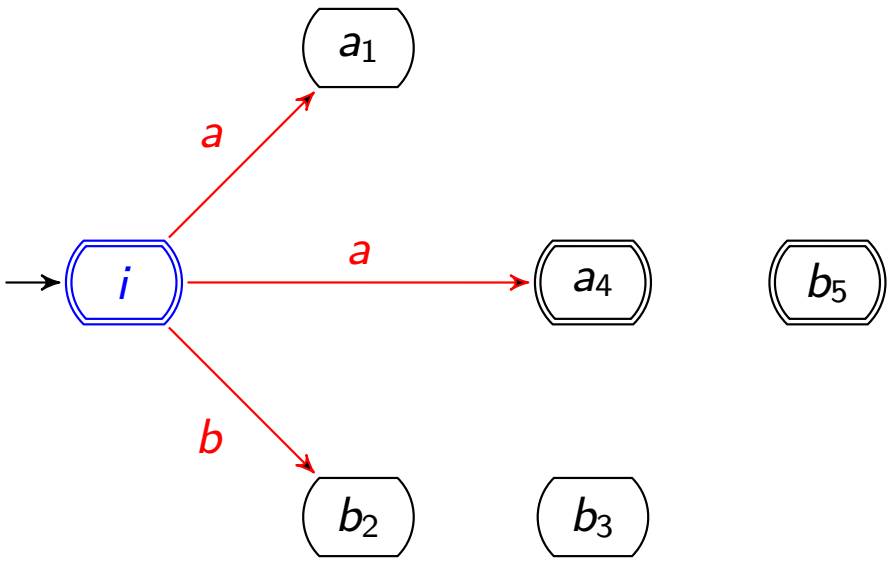

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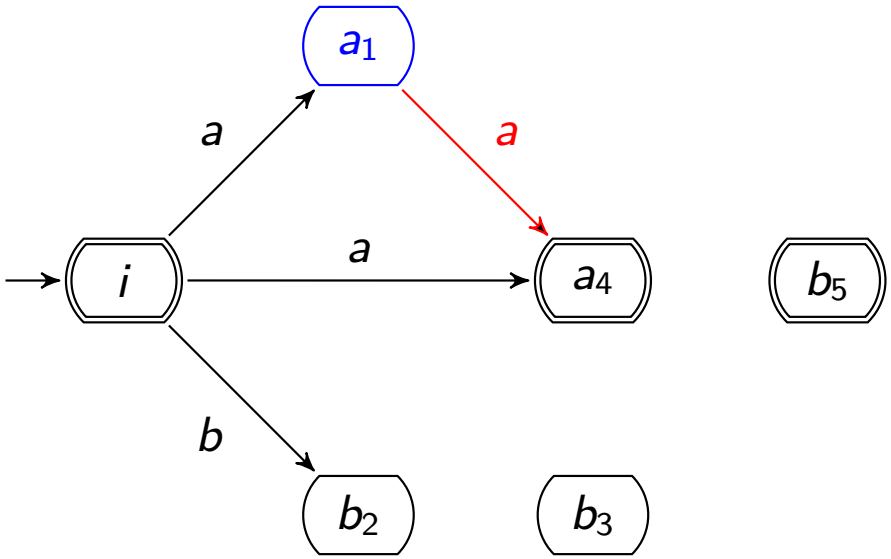

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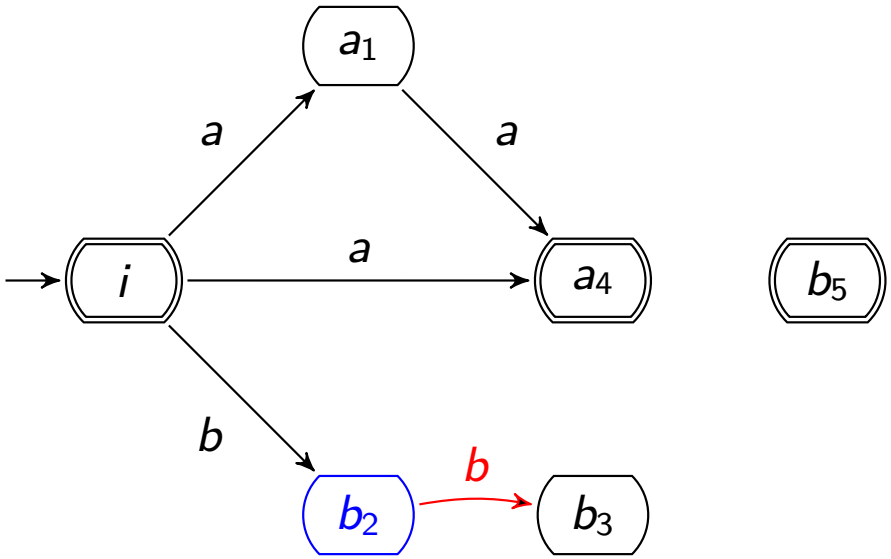

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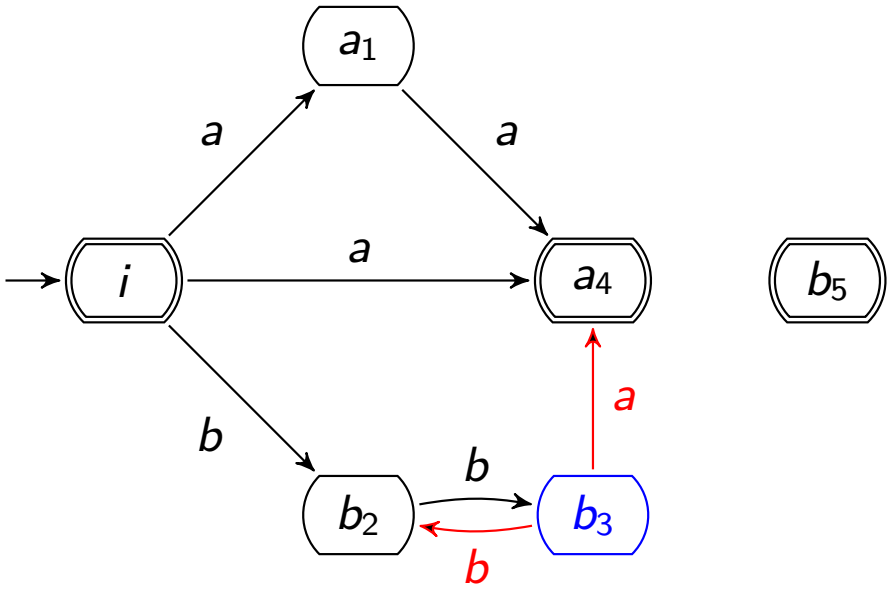

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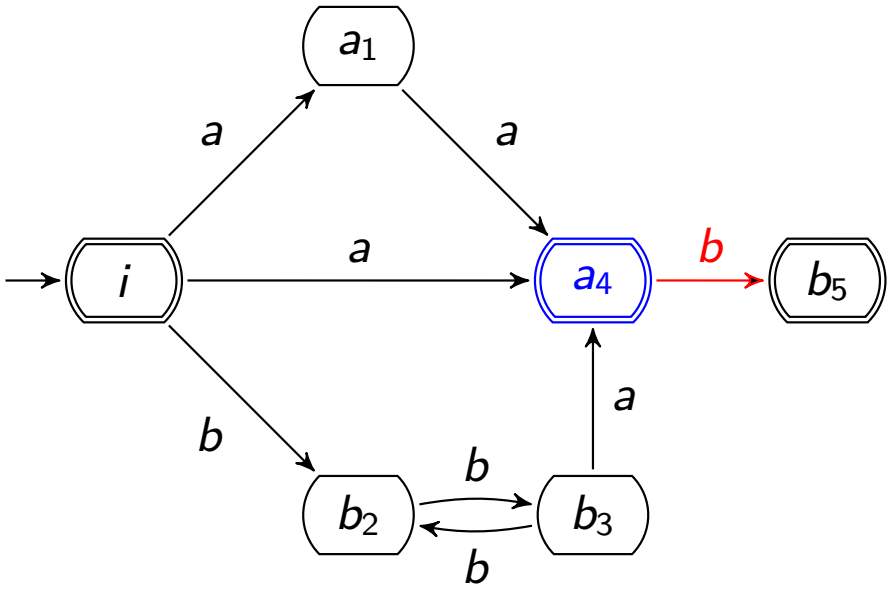

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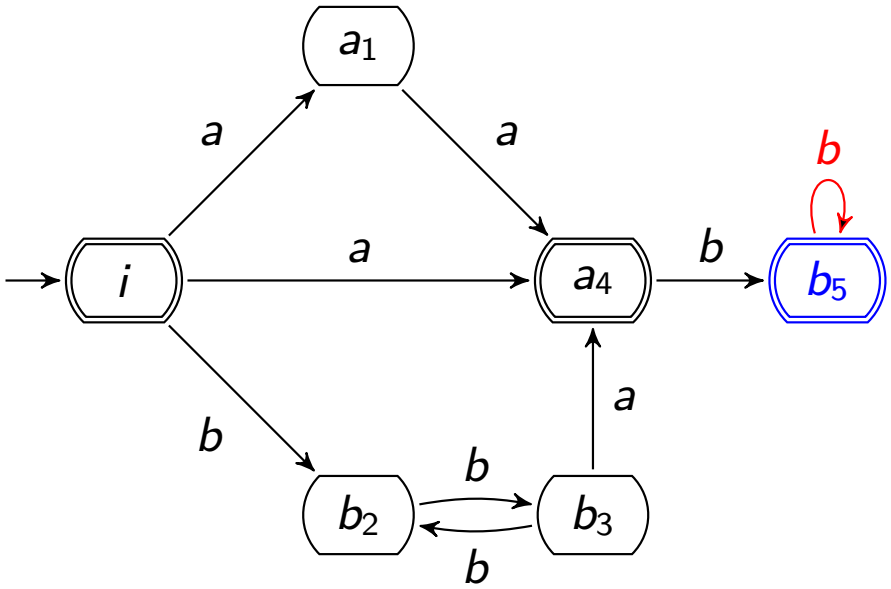

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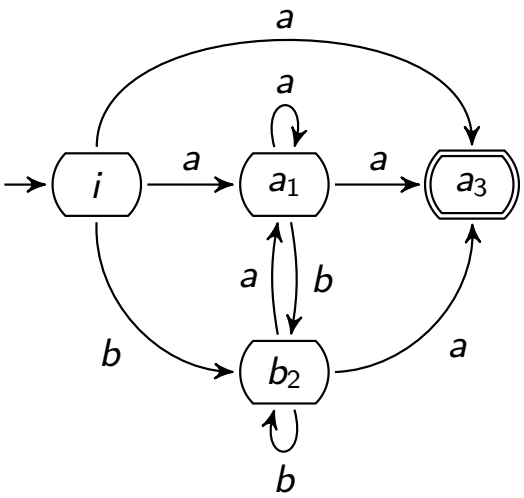
1-unambiguous regular expression

A regular expression E is 1-*unambiguous* if its Glushkov automaton is deterministic.



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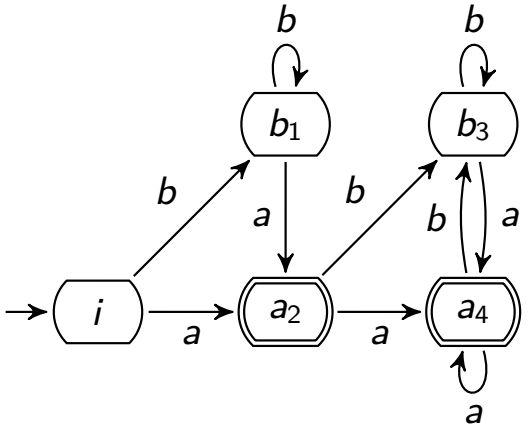

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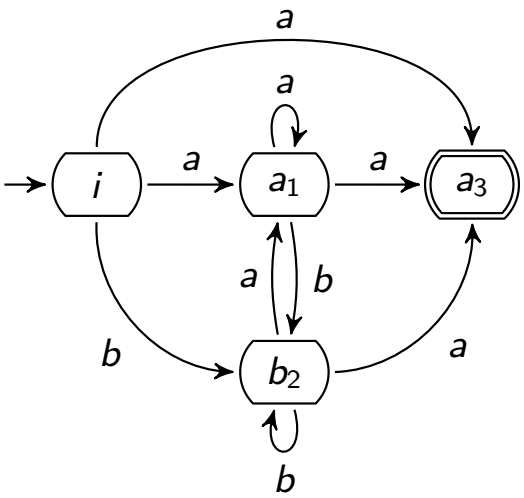

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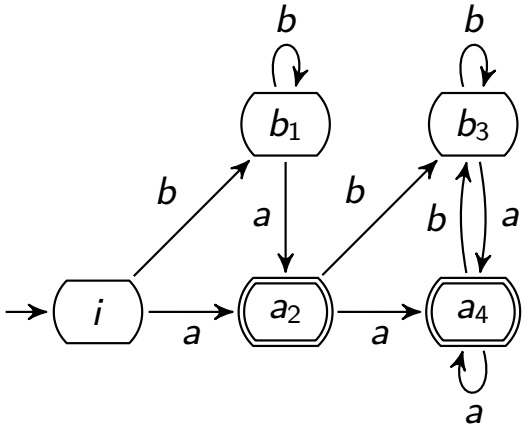

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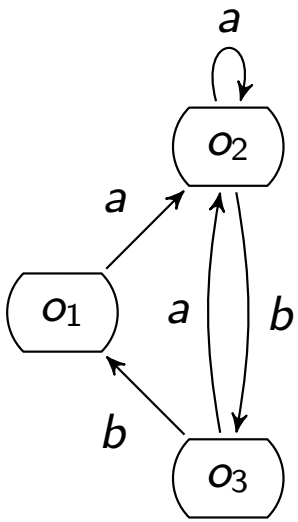
Orbit, gate and orbit property



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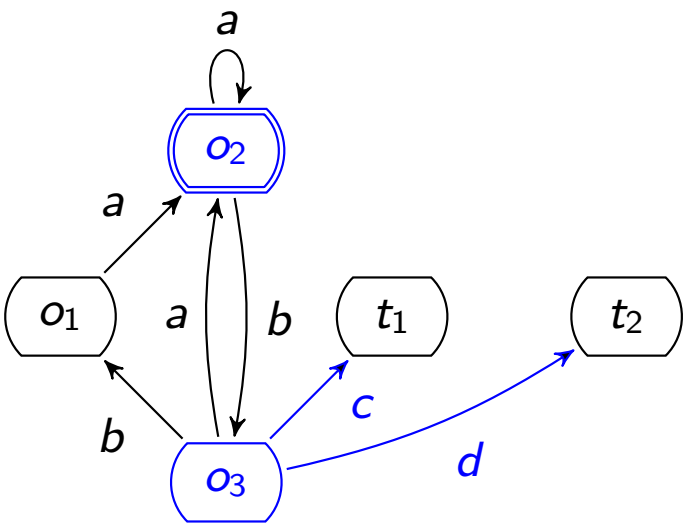

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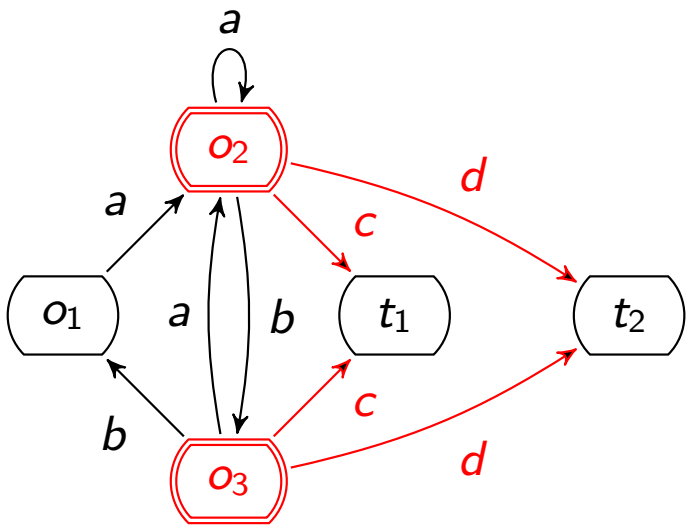

Outerly transverse orbit

All the gates have the same finality and the same transitions to outside states.

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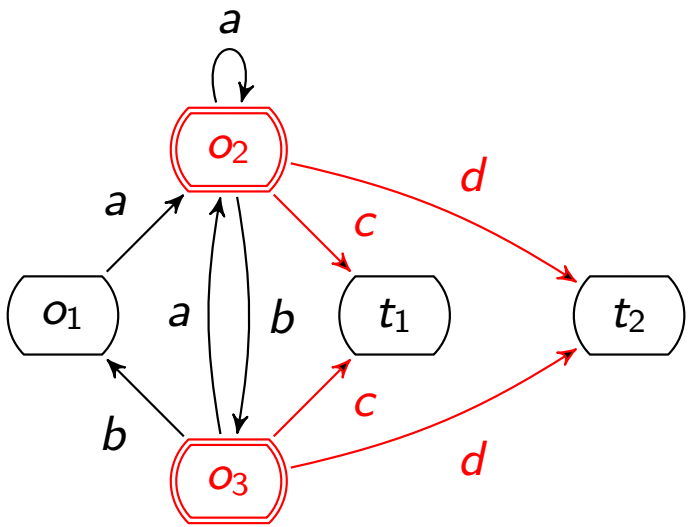

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Orbit property

An automaton has the orbit property if all of its orbits are outerly transverse.

Determining whether a language L is 1-unambiguous

A necessary condition

If a language is 1-unambiguous, then its minimal DFA has the orbit property.

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If a language is 1-unambiguous, then its minimal DFA has the orbit property.

Brüggemann-Klein and Wood

There exist regular languages which are not 1-unambiguous

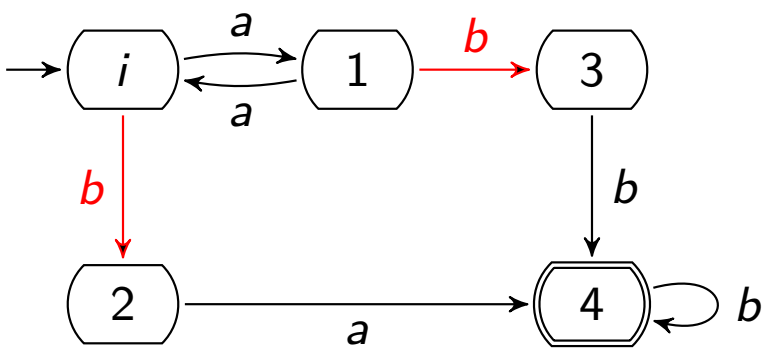

Figure: The minimal DFA recognizing $(aa)^*(abb + ba)b^*$

k-block regular expression and k-block automaton

- a block = a non-empty word
- k-block: when all blocks have a length at most k
- blocks can be considered as symbols in a different alphabet

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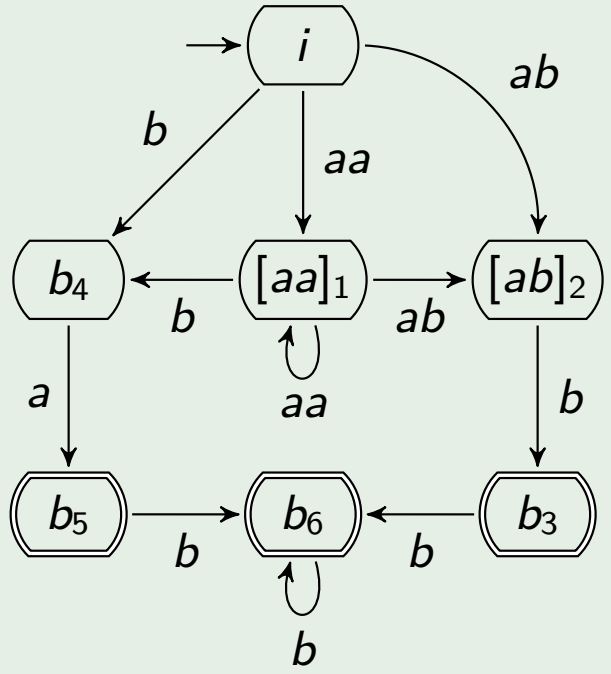
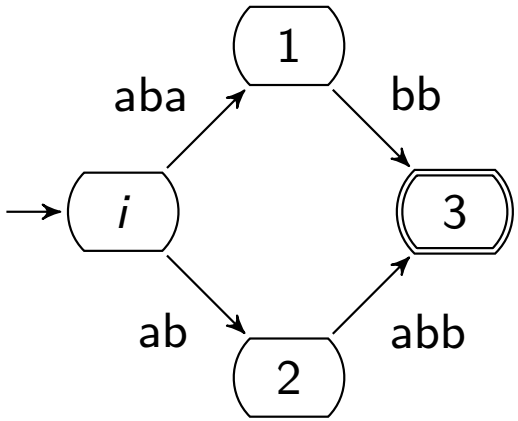


Block deterministic automaton

Block determinism

A block automaton $B = (\Gamma, Q, I, F, \delta)$ is *block deterministic* if:

- it is deterministic



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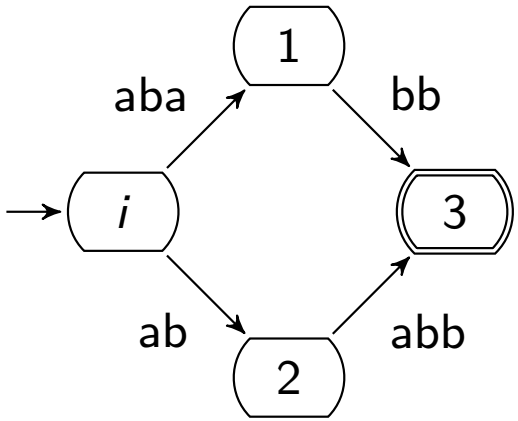

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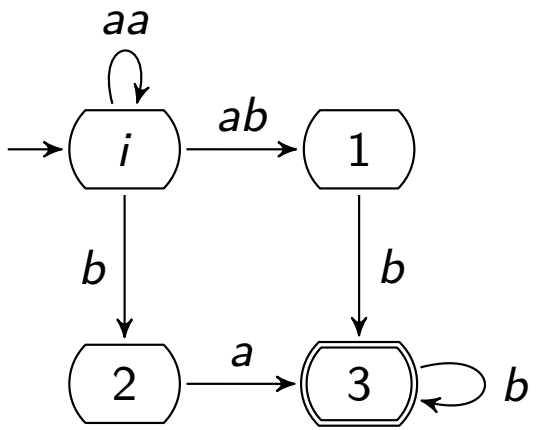

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k-block deterministic regular languages

Definitions

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- A regular language is k-block deterministic if it can be denoted by a k-block deterministic regular expression.

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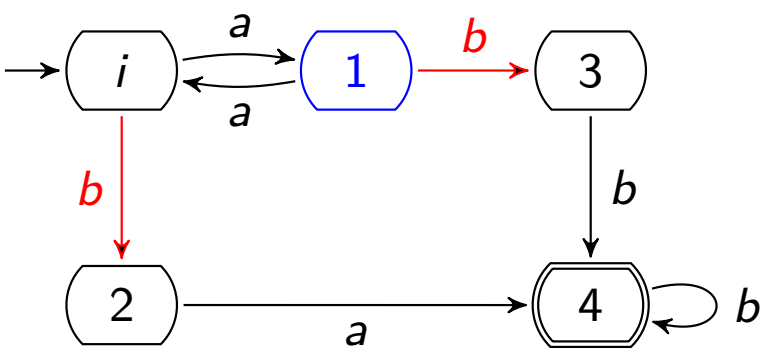

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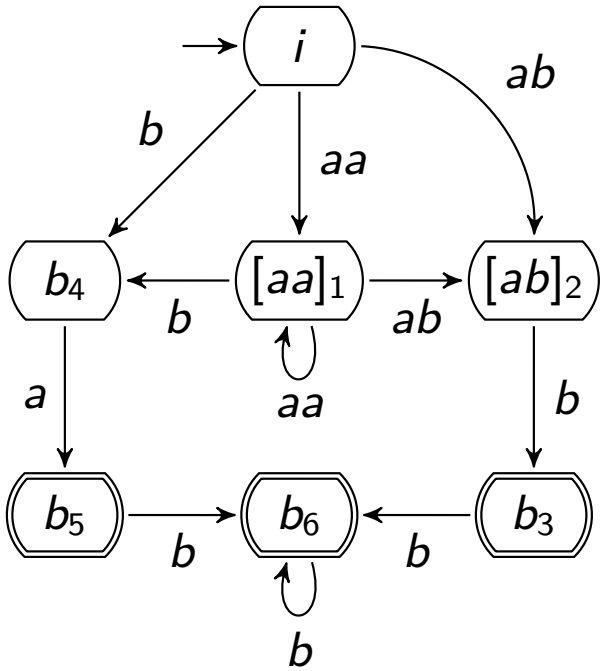

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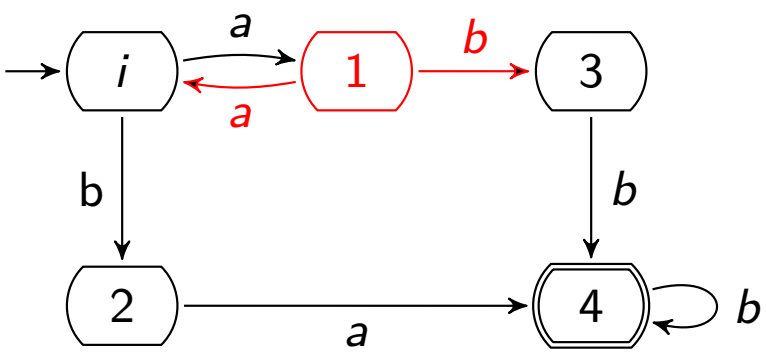
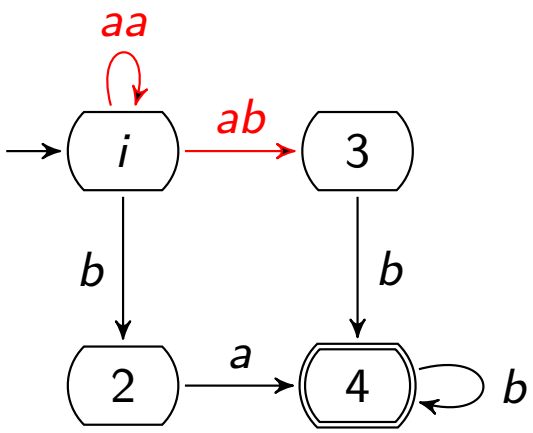
State elimination on block automaton





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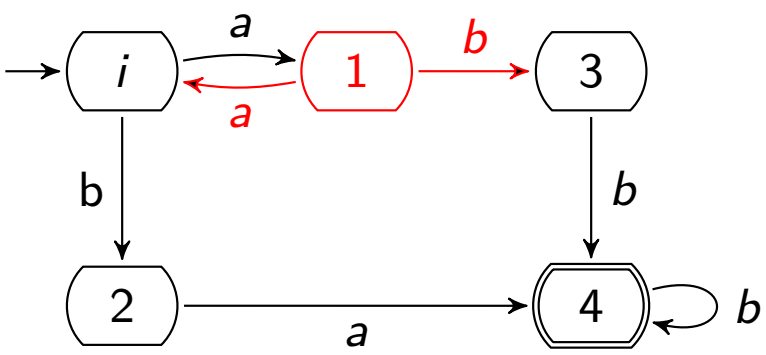
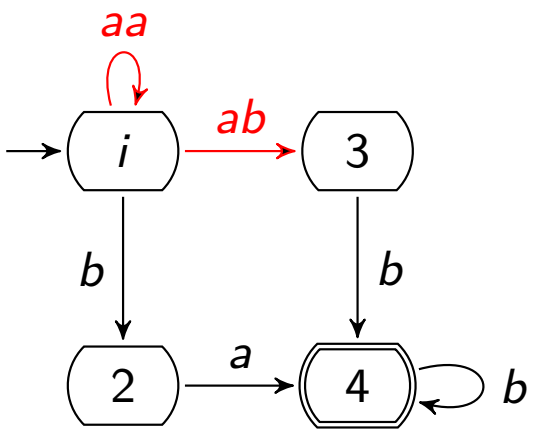
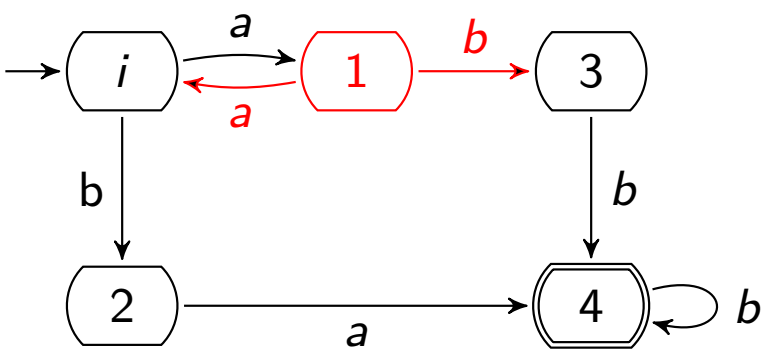


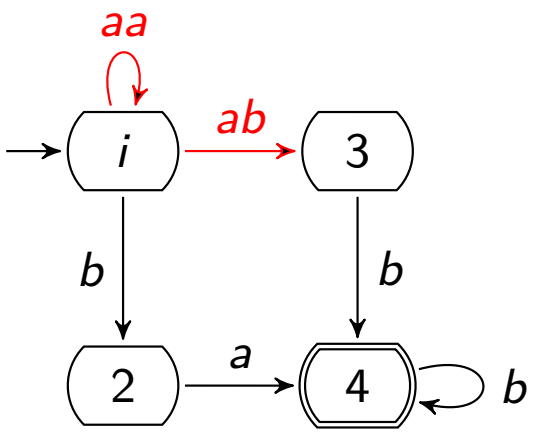


State elimination precondition

A state satisfies the *state elimination precondition* if it is neither initial nor final and it has no self-loop.

State elimination on block automaton





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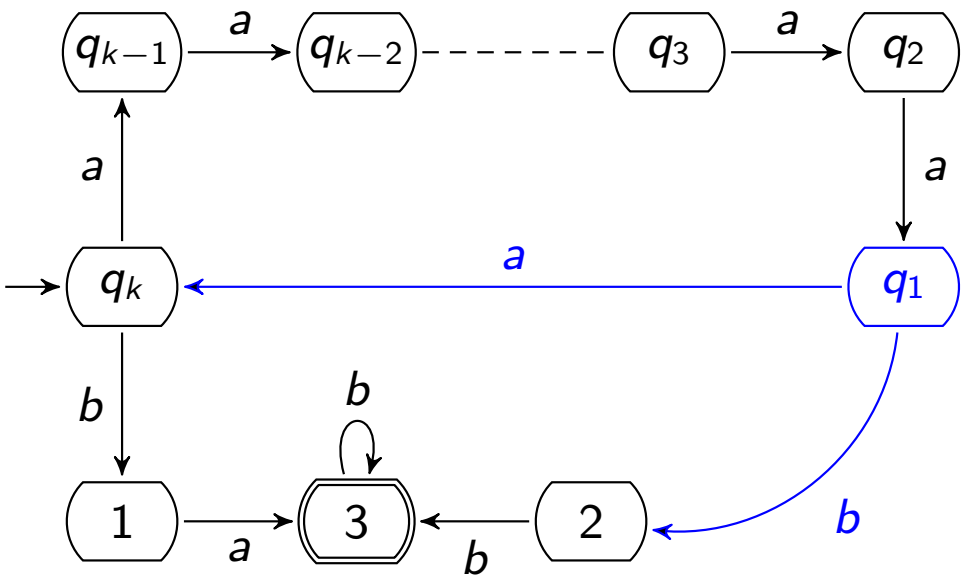
State elimination lemma

If a language is k-block deterministic, then, from the minimal DFA, we can get a k-block deterministic automaton with the orbit property using only state elimination.

Han and Wood parameterized family: HW_k



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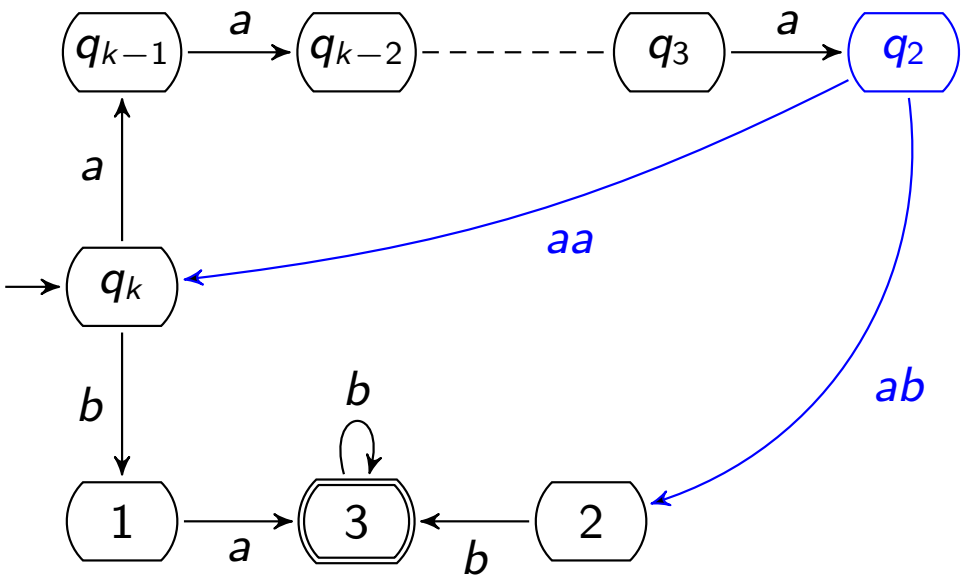

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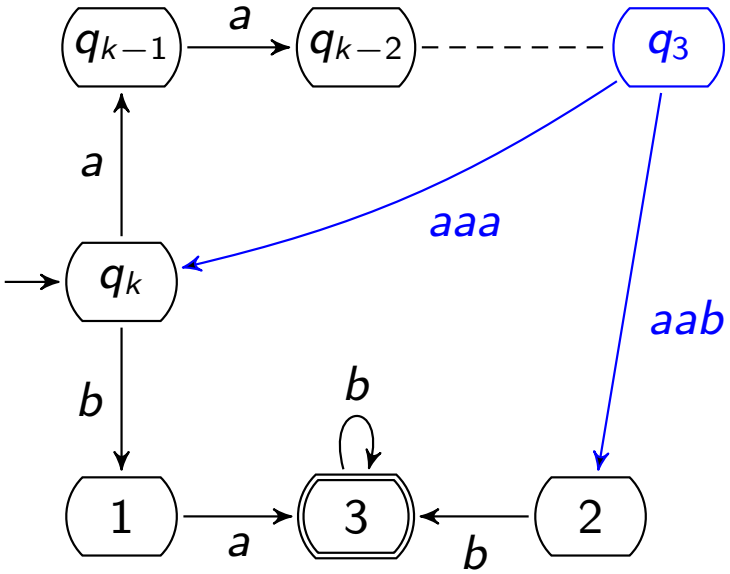

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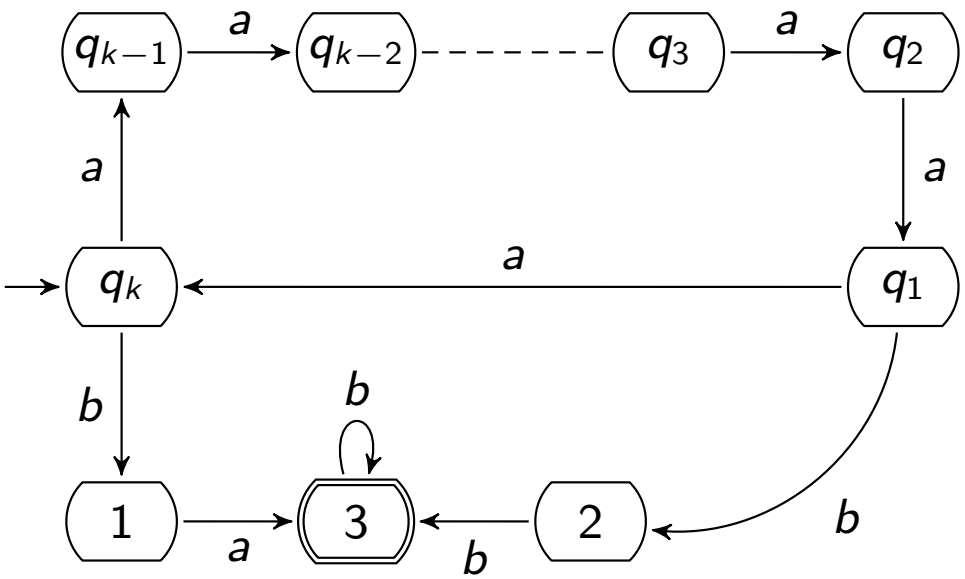

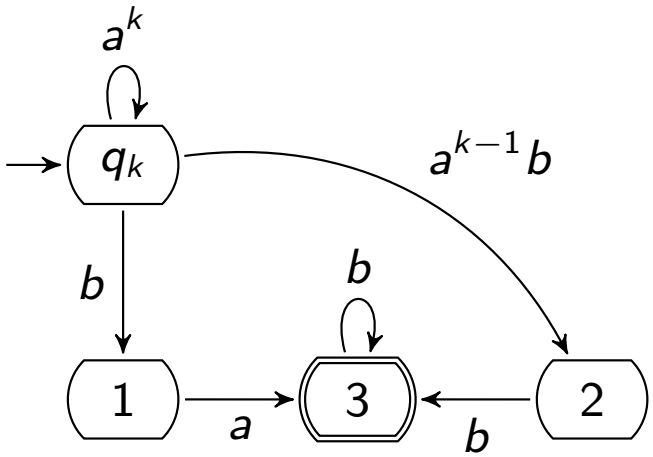

The family of languages HW_k can be denoted by $E_k = [a^k]^*([a^{k-1}b]b + ba)b^*$.

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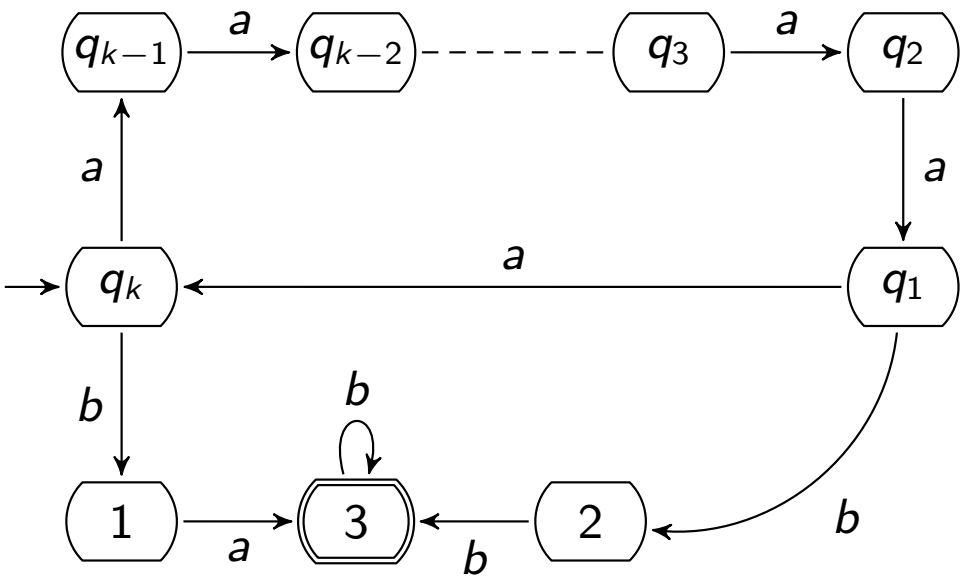
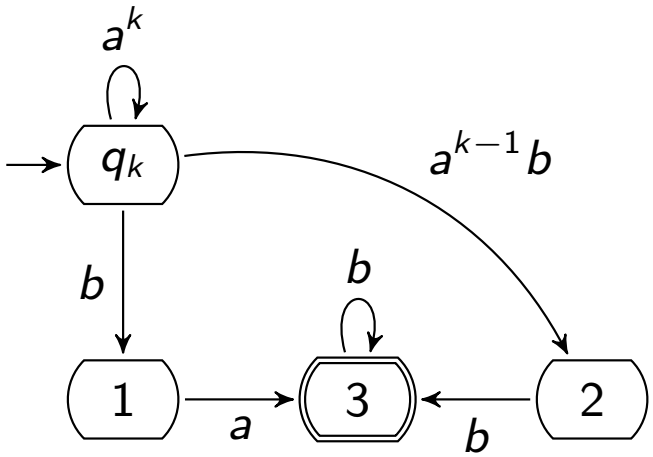




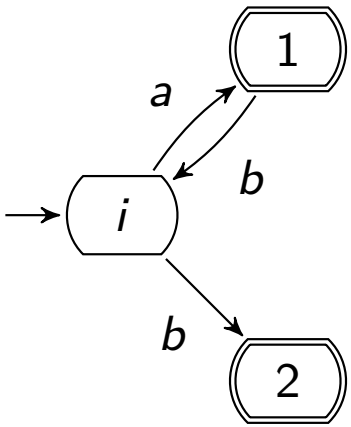
The family of languages HW_k can be denoted by $E_k = [a^k]^*([a^{k-1}b]b + ba)b^*$.

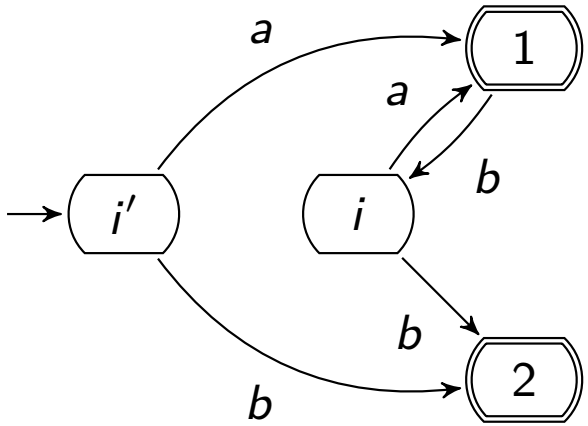
It is not $(k-1)$ since we have to eliminate states q_1 to q_{k-1} to obtain the orbit property.

A counter-example for the lemma

Standard automaton

An automaton $A = (\Sigma, Q, I, F, \delta)$ is *standard* if $|I| = 1$ and $\forall (p, a, q) \in \delta, q \notin I$.





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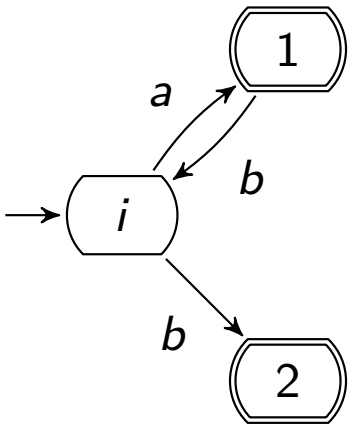

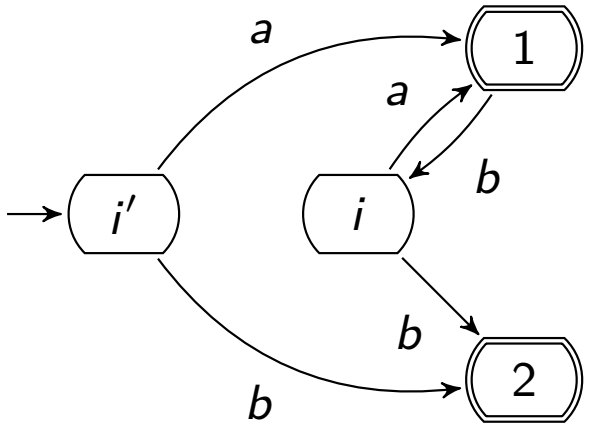

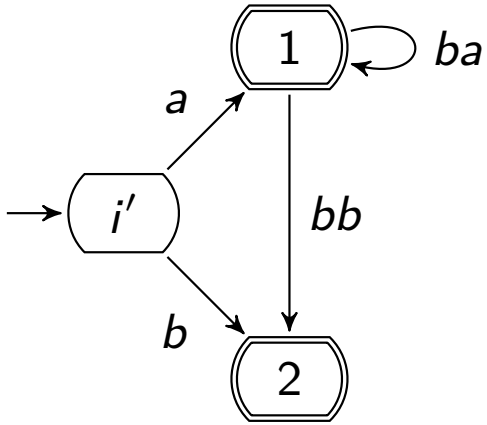

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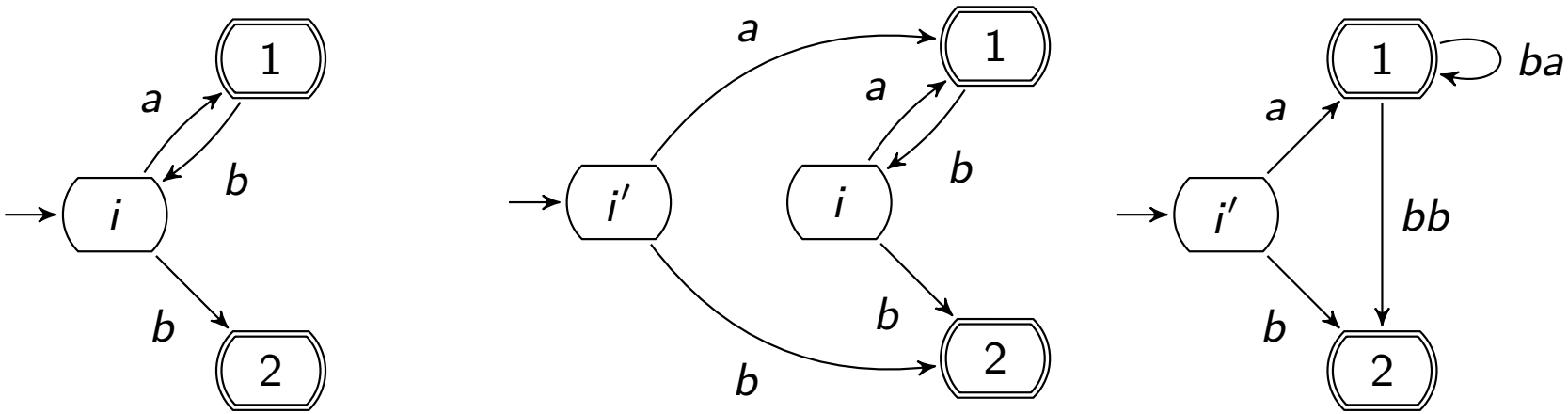
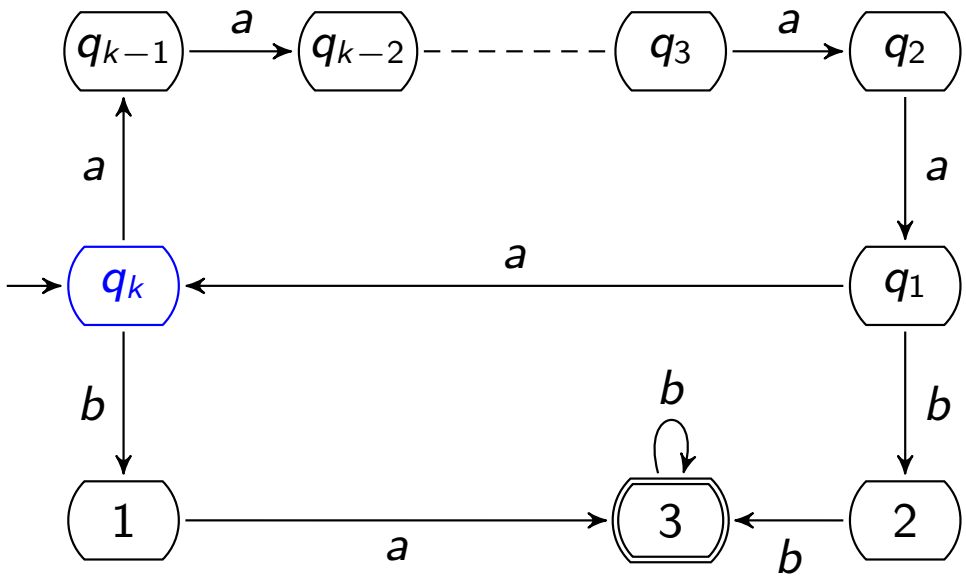


State elimination lemma is erroneous

State elimination alone from the minimal DFA is not sufficient to determine if a regular language is block deterministic.

HW_k is 2-block deterministic for all $k \geq 2$



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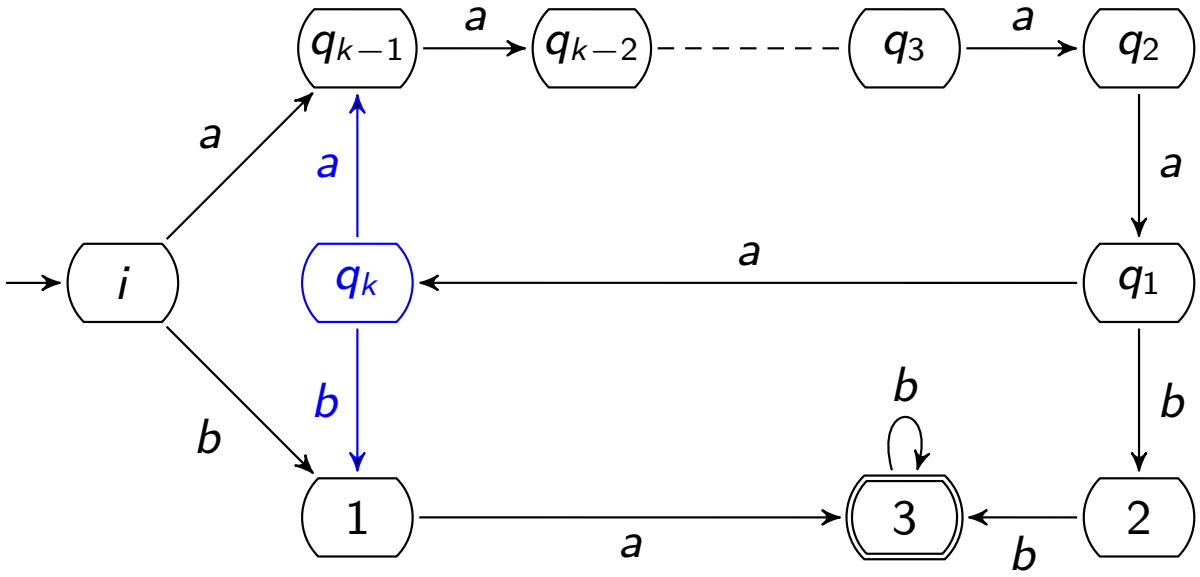

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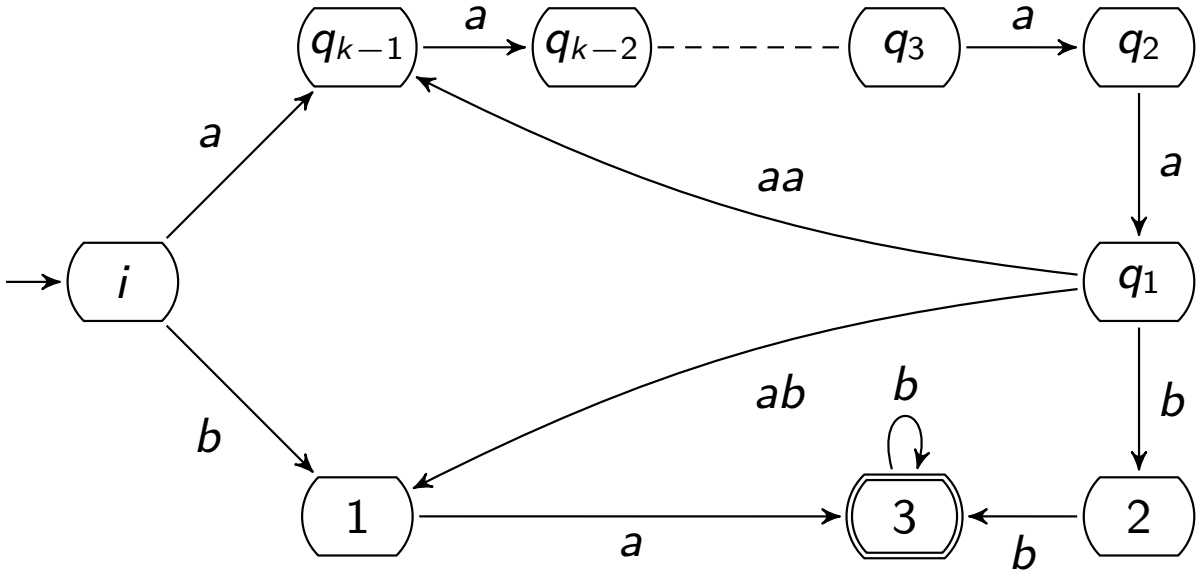
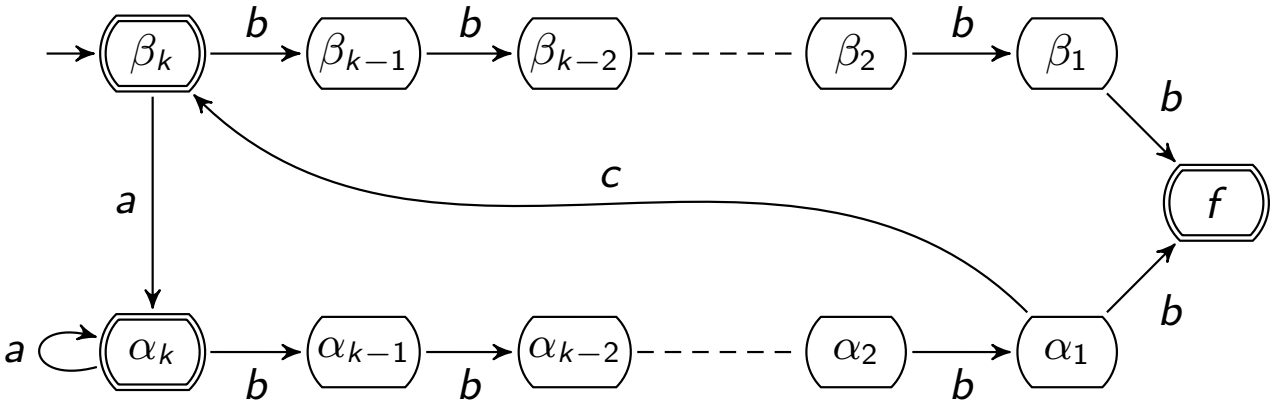


The family of languages HW_k can be denoted by
$F_k = (a^{k-1}([aa]a^{k-2})^*([ab]a + bb) + ba)b^*$

Our parameterized family: CMM_k



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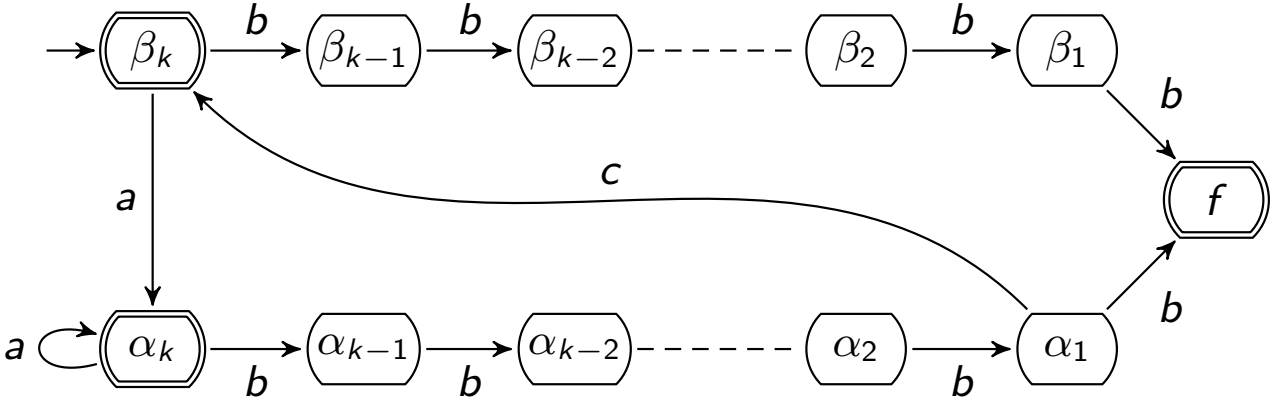

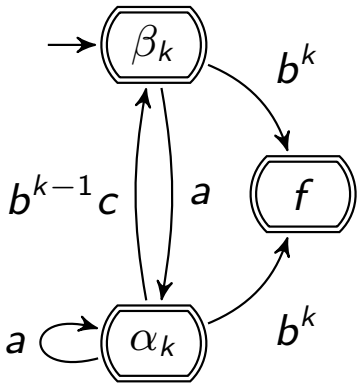

The family of languages CMM_k denoted by $E_k = (a(\varepsilon + [b^{k-1}c]))^*(\varepsilon + [b^k])$

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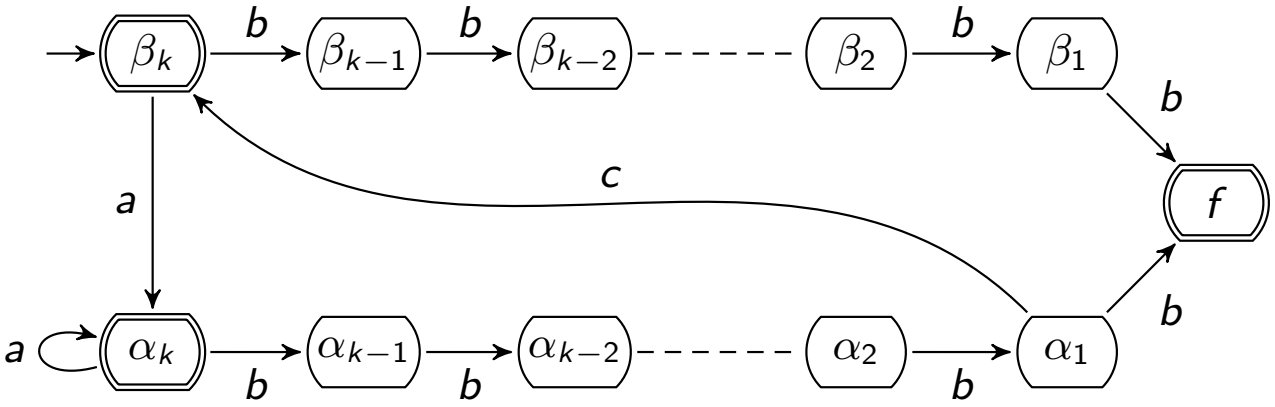

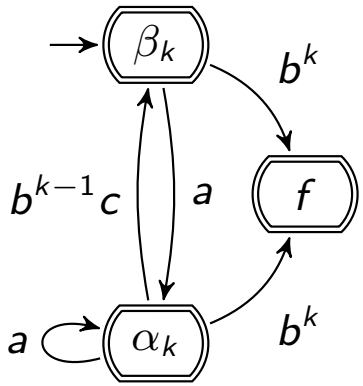

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Why is CMM_k not $(k-1)$-block deterministic: some ideas of the proof

- If CMM_k is $(k-1)$-block deterministic, then it is recognized by a $(k-1)$-block deterministic automaton with the orbit property.

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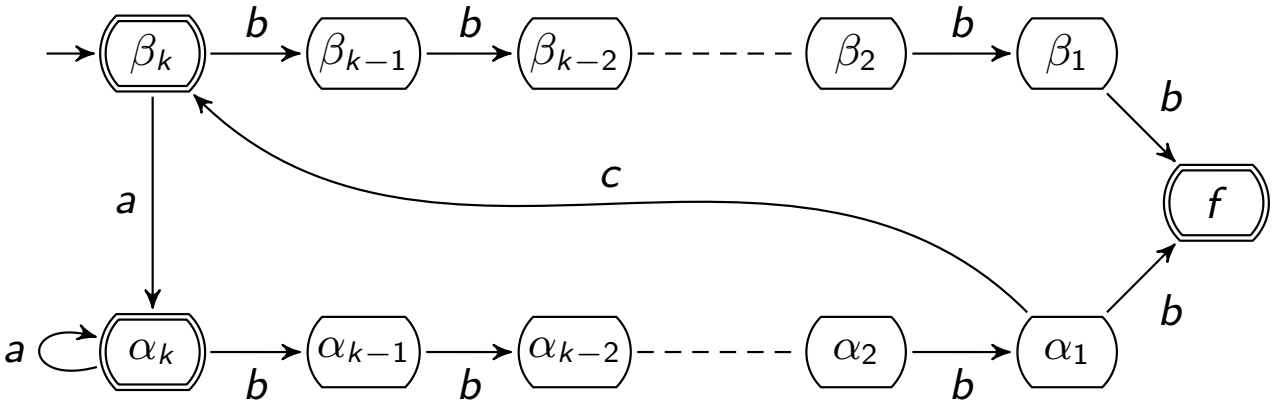

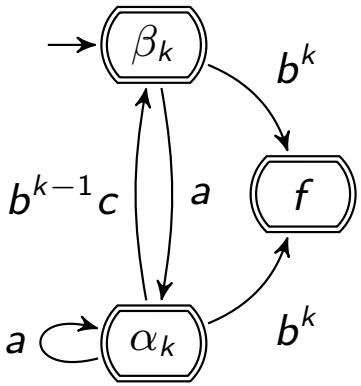

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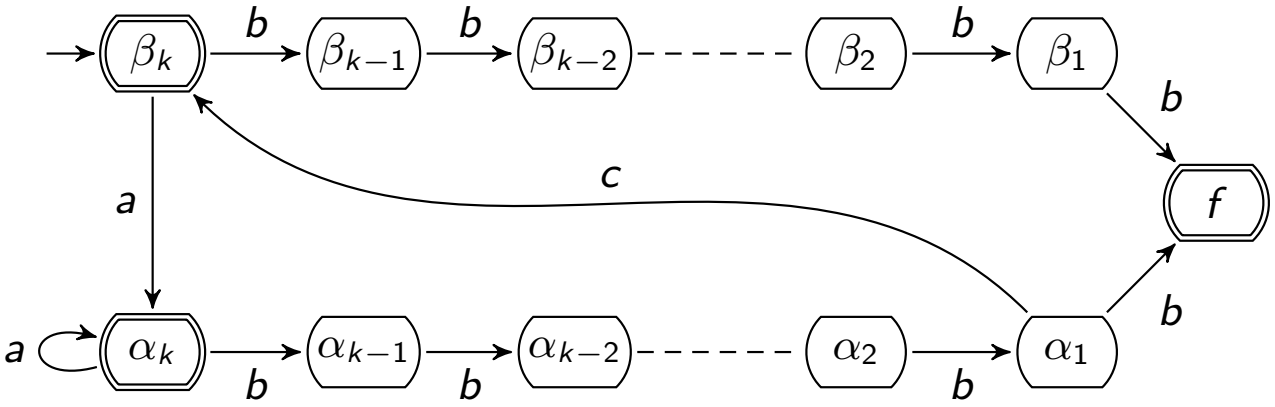

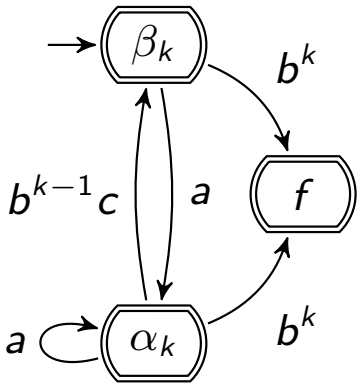

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- In this orbit, the orbit property cannot be obtained with blocks of length $k-1$.

Thank you for your attention

Do you have any question?